<u>Def</u> If $p$ and $q$ are statement variables, the conditional of $q$ by $p$ is "If $p$ then $q$" or "$p$ implies $q$" and denoted $p \to q$. It is false when $p$ is true and $q$ is false; otherwise it is true.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

<u>Ex</u>: Use a truth table to show that $p \lor q \to r \equiv (p \to r) \land (q \to r)$.

| $p$ | $q$ | $r$ | $p \lor q$ | $p \to r$ | $q \to r$ | $p \lor q \to r$ | $(p \to r) \land (q \to r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | | | | | |
| T | T | F | | | | | |
| T | F | T | | | | | |
| T | F | F | | | | | |
| F | T | T | | | | | |
| F | T | F | | | | | |
| F | F | T | | | | | |
| F | F | F | | | | | |

<u>Ex</u>: Rewrite the following statement in *if-then* form: **You pay tuition or you can't attend classes.**

Observation from the previous example: $p \to q \equiv \sim p \lor q$

Construct a truth table to verify the logical equivalence of the above observation:

| $p$ | $q$ | $\sim p$ | $p \to q$ | $\sim p \lor q$ |
|---|---|---|---|---|
| T | T | | | |
| T | F | | | |
| F | T | | | |
| F | F | | | |

<u>Ex</u>: Use logic operations to find a statement equivalent to the negation of a conditional statement.

From the example above:

$p \to q \equiv \sim p \lor q$

<u>Ex</u>: Write the negation of the following statement: **If it is May 22, then we have the Final Exam.**

Negation:

DEF The __contrapositive__ of a conditional statement of the form "If $p$ then $q$" is __If $\sim q$ then $\sim p$.__

i.e. The __contrapositive__ of $p \to q$ is __$\sim q \to \sim p$__

Important: A conditional statement is __logically equivalent__ to its contrapositive. [Homework problem will show.]

EX: Write the contrapositive of the following statement: **If I go to the gym, then I stretch.**

Contrapositive:

DEF Given the conditional statement "If $p$ then $q$",

- The CONVERSE is __"If $q$ then $p$."__ i.e. The converse of $p \to q$ is __$q \to p$__ .
- The INVERSE is __"If $\sim p$ then $\sim q$."__ i.e. The inverse of $p \to q$ is __$\sim p \to \sim q$__ .

Important: The converse and inverse are __NOT logically equivalent__ to the original conditional statement.

But, they are logically equivalent to __each other__ since they are the __contrapositive of each other.__

EX: Write the converse and inverse of the following statement: **If I go to the gym, then I stretch.**

Converse:

Inverse:

The statement "$p$ only if $q$" means __"If not $q$ then not $p$."__ (symbolically: __$\sim q \to \sim p$__

which is equivalent to __$p \to q$__ since it is the __contrapositive__ .

Be careful, it is not saying __"$q$ implies $p$"__ .

The expression __"$p$ if $q$"__ is really $q \to p$.

EX: Use the contrapositive to rewrite the following in 2 ways: **She will move only if the job offer is good.**

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

<u>Def</u> The biconditional of $p$ and $q$ is "$p$ if, and only if q", denoted $p \leftrightarrow q$. It is true if both $p$ and $q$ have the same truth values and false otherwise.

Note: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ [See book for truth table.]

<u>Def</u> $r$ is a sufficient condition for $s$ means "if $r$, then $s$". i.e., $r \to s$

<u>Def</u> $r$ is a necessary condition for $s$ means "if not $r$, then not $s$". i.e., $\sim r \to \sim s$

Which has the equivalent contrapositive: "if $s$, then $r$" $s \to r$.

<u>Def</u> $r$ is a necessary and sufficient condition for $s$ means "$r$ iff $s$". i.e., $\sim r \leftrightarrow \sim s$

<u>Ex</u>: If you travel internationally, then you have a passport.

Rewrite using "sufficient":

Rewrite using "necessary":

<u>Ex</u>: Rewrite the following statement in "if-then" form in 2 ways using the contrapositive.

**Having 2 sides of equal length is necessary for a triangle to be isosceles.**

1.

2.