Name: Key

Quiz 2

Math 301 Discrete Mathematics – Crawford

6 March 2019

Books, calculators, and notes (in any form) are not are allowed. Show all your work for credit. ***Good luck!***

1. (8 pts) State whether the following arguments are valid or invalid. If it is a valid argument, state whether the form is modus ponens or modus tollens. If it is an invalid argument, state whether it exhibits the converse error or the inverse error.

(a). All safe drivers obey the traffic laws.
Mayhem is not a safe driver.
$\therefore$ Mayhem does not obey traffic laws.

If $P(x)$ then $Q(x)$
$\sim P(a)$
$\therefore \sim Q(a)$

Invalid: Inverse Error

(b). All Jedi use the Force.
Rey uses the Force.
$\therefore$ Rey is a Jedi.

If $P(x)$ then $Q(x)$
$Q(a)$
$\therefore P(a)$

Invalid: Converse Error

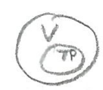
(c). All tourists in Chicago eat deep dish pizza
Jill did not eat deep dish pizza.
$\therefore$ Jill is not a tourist in Chicago.

If $P(x)$ then $Q(x)$
$\sim Q(a)$
$\therefore \sim P(a)$

Valid: Modus tollens

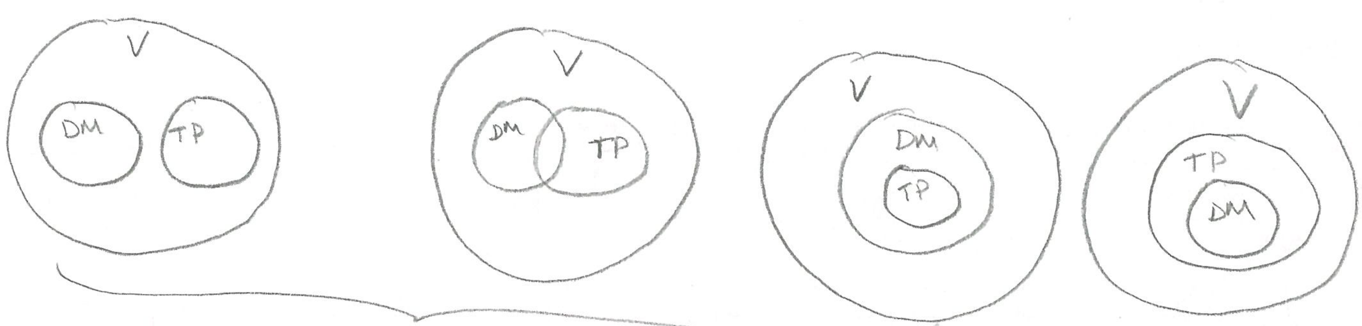
2. (4 pts) Indicate whether the following argument is valid or invalid. Support your answer by drawing diagrams.

All discrete mathematics students can tell a valid argument from an invalid one.


All thoughtful people can tell a valid argument from and invalid one.


$\therefore$ All discrete mathematics students are thougthful.

Let DM be the set of all discrete math students.
Let TP " " " " thoughtful people
Let V " " " " people who can tell a valid argument from an invalid one.



In these 3 cases, the premises are true, but the conclusion is false.

Invalid

3. (4 pts) Fill in the blanks to the following proof.

THEOREM Whenever n is an odd integer, $3n^2 + 5$ is even.

PROOF Suppose n is any odd integer. [Show that $3n^2+5$ is even.]

By definition of odd, $n =$ $2k+1$ for some integer k.

Then

$$3n^2 + 5 = 3(2k+1)^2+5$$
$$= 3(4k^2 + 4k + 1) + 5$$
$$= 12k^2 + 12k + 3 + 5$$
$$= 12k^2 + 12k + 8$$
$$= 2(6k^2+6k+4)$$
$$= 2s \text{ where } s = 6k^2+6k+4,$$

And s is an integer since sums & products of integers are integers,

i.e., $3n^2 + 5 = 2s$, for integer s.

Therefore by definition, $3n^2 + 5$ is even. ■

4. (2 pts) Disprove the following statement by giving a counterexample:

For all integers n, if n is prime then $(-1)^n = -1$.

Let $n=2$ which is prime. But $(-1)^2 = 1 \neq -1$

5. (2 pts) Prove:

There are distinct integers a and b such that $\frac{1}{a} + \frac{1}{b}$ is an integer.

$\exists$

Let $a=1$ and $b=-1$ which are 2 distinct integers.

Then $\frac{1}{a}+\frac{1}{b} = \frac{1}{1}+\frac{1}{-1} = 1-1 = 0$ which is an integer.