Name: _____ **Exam 2**

Math 301 Discrete Mathematics – Crawford **1 May 2019**

- Books, notes (in any form), and calculators are not allowed.
- ***Put all of your work and answers on other sheets of paper.*** Include this sheet as a cover sheet.
- ***Show all your work***. Partial credit may be given for written work.

Good Luck!

	Score
1	/12
2	/12
3	/12
4	/10
5	/14
6	/14
7	/14
8	/14
Total	/100

1. (12 pts). Given the following algorithm, make a trace table and clearly state the final values of j, s, and t.

$j := 3$
$s := 18$
$t := 4$
while $j \neq 7$
    if ($j > 5$ or $j = 3$)
        then $s := s - 3$
        else $t := 2t + j$
    $j := j + 1$
end while

2. (12 pts). Let $a_0 = -1, a_1 = 2, a_2 = -2, a_3 = 3, a_4 = -2, a_5 = 2$, and $a_6 = -1$. Compute each of the following:

(a). $\sum_{k=0}^{6} a_k$ **(b).** $\sum_{j=1}^{3} a_{2j}$ **(c).** $\prod_{i=0}^{3} a_i$ **(d).** $\prod_{k=1}^{3} k^2$

3. (12 pts). Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{u, v, w, x, y\}$ and define $h : X \to Y$ as follows:

$$h(1) = v,\ h(2) = x,\ h(3) = v,\ h(4) = v,\ h(5) = y.$$

(a). Draw an arrow diagram for h.

(b). Let $A = \{1, 2\}, C = \{x, v\}, D = \{w\}$. Find $h(A), h^{-1}(C), h^{-1}(D)$.

4. (10 pts). Define a relation P on $\mathbb{Z}$ as follows: For every ordered pair $(m, n) \in \mathbb{Z} \times \mathbb{Z}$,

$m\ P\ n$ iff m and n have a common prime factor. [Justify your answers.]

(a). Is $15\ P\ 25$? **(b).** Is $0\ P\ 5$? **(c).** Is $22\ P\ 27$?

5. (14 pts). Define $g : \mathbb{Z} \to \mathbb{Z}$ by the rule $g(n) = 2n + 5$, for each integer n.

(a). Is g one-to-one? Prove or give a counterexample.

(b). Is g onto? Prove or give a counterexample.

6. (14 pts). Prove by contradiction: For any even integer n, $n^2 - 2$ is not divisible by 4.

7. (14 pts). Prove by Mathematical Induction:

For every integer $n \geq 1$, $\quad 1 + 6 + 11 + 16 + \cdots (5n - 4) = \dfrac{n(5n - 3)}{2}$

8. (14 pts). Prove: For all sets A, B, and C, $\quad (A - B) \cup (C - B) \subseteq (A \cup C) - B$.