Name: Key **Exam 1**

Math 301 Discrete Mathematics – Crawford **13 March 2019**

Books, calculators, and notes (in any form) are not are allowed. Show all your work for credit. ***Good luck!***

1. (12 pts) Write the negation of the following statements.

(a). The variable S is undeclared and the data are out of order.

The variable S is declared or the data are in order.

or not undeclared

or not out of order

(b). All daydream believers are homecoming queens.

There is a daydream believer who is not a homecoming queen.

or Some daydream believers are not homecoming queens.

(c). For all integer n, if n is prime then n is odd or n is 2.

There exists an integer n, such that n is prime and n is even and $n \neq 2$.

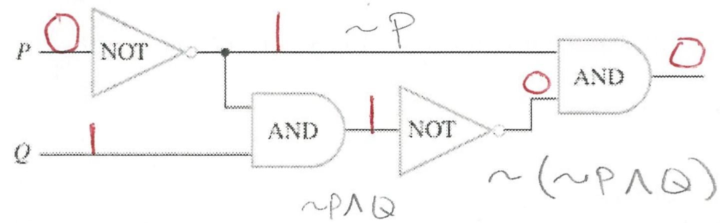
2. (10 pts) Use a truth table to determine if the following two statements are logically equivalent. Include a sentence explaining your conclusion.

$$p \to q \vee r, \qquad p \wedge {\sim}r \to q$$

p	q	r	$q \vee r$	$p \to q \vee r$	$\sim r$	$p \wedge \sim r$	$p \wedge \sim r \to q$
T	T	T	T	T	F	F	T
T	T	F	T	T	T	T	T
T	F	T	T	T	F	F	T
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	T	T	F	T
F	F	T	T	T	F	F	T
F	F	F	F	T	T	F	T

The statements have the same truth values. Therefore, they are logically equivalent.

3. (10 pts) Given the circuit below.



$\sim P \wedge (\sim(\sim P \wedge Q))$

(a). Determine the output signal S if the input signals are $P = 0$ and $Q = 1$. [Show the input, intermediate, and output signals on the diagram above.]

$\boxed{S = 0}$

(b). Find the Boolean expression that corresponds to the circuit.

$\boxed{\sim P \wedge (\sim(\sim P \wedge Q))}$

4. (14 pts) Given the following statement: $\forall$ real numbers x, if $x^2 \geq 1$, then $x > 0$. $\boxed{\text{False}}$

$x = -2$
$x^2 = 4 \geq 1$ but $x \not> 0$

(a). Write its contrapositive, converse, and inverse.

Contrapositive:

$\boxed{\forall \text{ real numbers } x, \text{ if } x \leq 0 \text{ then } x^2 < 1.}$ $\boxed{\text{False}}$

Converse:

$\boxed{\forall \text{ real numbers } x, \text{ if } x > 0 \text{ then } x^2 \geq 1}$ $\boxed{\text{False}}$

$x = \frac{1}{2} > 0$
but $x^2 = \frac{1}{4} \not\geq 1$

Inverse:

$\boxed{\forall \text{ real numbers } x, \text{ if } x^2 < 1 \text{ then } x \leq 0}$ $\boxed{\text{False}}$

(b). Indicate which of the above (statement, contrapositive, converse, and inverse) are true and which are false.

See above. Counterexamples not required.

5. (12 pts) A group of students attend a book fair in which there are 4 tables each with a different genre of free books. Each student can choose up to 6 books from these 4 genres. The available genres and books are as follows:

SELF-HELP: It's Not You; Adulting; BYE Student Loan Debt

SCIENCE FICTION: Dune; Ender's Game

AUTOBIOGRAPHIES: Born a Crime; Becoming

CLASSICS: Huckleberry Finn; To Kill a Mockingbird

The students chose the following books:

Chris: BYE Student Loan Debt; Dune; Born a Crime; Becoming; Huckleberry Finn

Jordan: BYE Student Loan Debt; Dune; Ender's Game; Huckleberry Finn; To Kill a Mockingbird

Riley: Adulting; BYE Student Loan Debt; Ender's Game; Becoming; To Kill a Mockingbird

Write the following statements *informally* and determine whether they are true or false (explain your reasoning).

(a). $\exists$ a book B such that $\forall$ students S, S chose B.

There is a book that was chosen by all students.

True since BYE Student Loan Debt was chosen by all students

(b). $\forall$ students S and $\forall$ genres G, $\exists$ a book B in G such that S chose B.

All students chose a book from each genre.

False, since Jordan did not choose an autobiography.

6. (12 pts) State whether the following arguments are valid or invalid. If it is a valid argument, state whether the form is modus ponens or modus tollens. If it is an invalid argument, state whether it exhibits the converse error or the inverse error.

(a). If the topological sorting is finite, then it has a minimal element.
The topological sorting is not finite.
∴ The topological sorting does not have a minimal element.

If p then q
$\sim p$
∴ $\sim q$

$\boxed{\text{Invalid; Inverse Error}}$

(b). The sum of any two rational numbers is rational.
The sum $r + s$ is rational.
∴ The numbers r and s are both rational.

If p then q
q
∴ p

If r, s are rational then the sum r+s is rational.

$\boxed{\text{Invalid; Converse Error}}$

(c). All dogs go to heaven.
Sparky did not go to heaven.
∴ Sparky is not a dog.

If $P(x)$ then $Q(x)$.
$\sim Q(a)$
∴ $\sim P(a)$

$\boxed{\text{Valid; Modus Tollens}}$

7. (8 pts)

(a). Prove the following existential statement: There are real numbers a and b such that $a < b$ and $a^2 \not< b^2$.

$\boxed{a = -2 \quad b = 1}$

So $a < b$ since $-2 < 1$ and $a^2 \not< b^2$ since $4 \not< 1$

(b). Disprove the following statement by giving a counterexample: The quotient of any two rational numbers is rational.

$\boxed{r = \frac{1}{2} \text{ and } s = 0}$ are both rational numbers. But $\frac{r}{s} = \frac{(\frac{1}{2})}{0}$ is undefined, and therefore not a rational number.

8. (12 pts) Prove the following statement directly from the definition(s).

If b is odd, then $b^2 - 6$ is odd.

Proof Let b be an odd integer. [Show b^2-6 is odd.]

Then $b = 2k+1$ for some integer k by definition of odd.

$b^2 - 6 = (2k+1)^2 - 6$ [Show $b^2-6 = 2m+1$]

$= 4k^2 + 4k + 1 - 6$

$= 4k^2 + 4k - 6 + 1$

$= 2(2k^2+2k-3)+1$

$= 2m+1$ where $m = 2k^2+2k-3$ is an integer since sums and products of integers are integers.

$\therefore$ b^2-6 is odd by definition. ∎

9. (12 pts) Prove the following statement directly from the definition(s).

The product of any two even integers is divisible by 4.

optional: If a and b are even integers, then $4|ab$

Proof

Let a and b be even integers. [Show $4|ab$]

Then by definition $a = 2m$ and $b = 2n$ for integers $m \neq n$.

By substitution $ab = (2m)(2n)$

$= 4mn$

$= 4k$ where $k = mn$ is an integer by the closure property of integers.

ab is also an integer by the closure properties of integers.

Then since $ab = 4k$ for some integer k, $4|ab$ by the definition of divisibility. ∎