Books, calculators, and notes (in any form) are <u>not</u> are allowed. Show all your work for credit. ***Good luck!***

1. (12 pts) Write the negation of the following statements.

(a). The variable S is undeclared and the data are out of order.

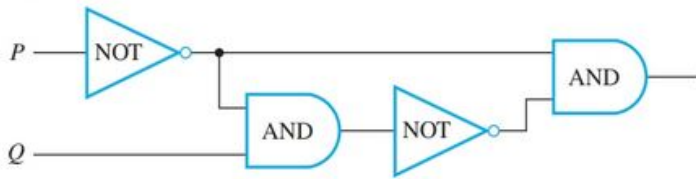**(b).** All daydream believers are homecoming queens.

(c). For all integer n, if n is prime then n is odd or n is 2.

2. (10 pts) Use a truth table to determine if the following two statements are logically equivalent. Include a sentence explaining your conclusion.

$$p \to q \lor r, \qquad p \land \sim r \to q$$

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

3. (10 pts) Given the circuit below,

(a). Determine the output signal S if the input signals are $P = 0$ and $Q = 1$. [Show the input, intermediate, and output signals on the diagram above.]

(b). Find the Boolean expression that corresponds to the circuit.

4. (14 pts) Given the following statement: $\forall$ real numbers x, if $x^2 \geq 1$ then $x > 0$.

(a). Write its contrapositive, converse, and inverse.

Contrapositive:

Converse:

Inverse:

(b). Indicate which of the above (statement, contrapositive, converse, and inverse) are true and which are false.

5. (12 pts) A group of students attend a book fair in which there are 4 tables each with a different genre of free books. Each student can choose up to 6 books from these 4 genres. The available genres and books are as follows:

SELF-HELP: It's Not You; Adulting; BYE Student Loan Debt

SCIENCE FICTION: Dune; Ender's Game

AUTOBIOGRAPHIES: Born a Crime; Becoming

CLASSICS: Huckleberry Finn; To Kill a Mockingbird

The students chose the following books:

Chris: BYE Student Loan Debt; Dune; Born a Crime; Becoming; Huckleberry Finn

Jordan: BYE Student Loan Debt; Dune; Ender's Game; Huckleberry Finn; To Kill a Mockingbird

Riley: Adulting; BYE Student Loan Debt; Ender's Game; Becoming; To Kill a Mockingbird

Write the following statements *informally* and determine whether they are true or false (explain your reasoning).

(a). $\exists$ a book B such that $\forall$ students S, S chose B.

(b). $\forall$ students S and $\forall$ genres G, $\exists$ a book B in G such that S chose B.

6. (12 pts) State whether the following arguments are valid or invalid. If it is a valid argument, state whether the form is modus ponens or modus tollens. If it is an invalid argument, state whether it exhibits the converse error or the inverse error.

(a).   If the topological sorting is finite, then it has a minimal element.
             The topological sorting is not finite.
        $\therefore$ The topological sorting does not have a minimal element.

(b).   The sum of any two rational numbers is rational.
             The sum $r + s$ is rational.
        $\therefore$ The numbers r and s are both rational.

(c).   All dogs go to heaven.
             Sparky did not go to heaven.
        $\therefore$ Sparky is not a dog.

7. (8 pts)

(a). Prove the following existential statement: There are real numbers a and b such that $a < b$ and $a^2 \not< b^2$.

(b). Disprove the following statement by giving a counterexample: The quotient of any two rational numbers is rational.

8. (12 pts) Prove the following statement directly from the definition(s).

If b is odd, then $b^2 - 6$ is odd.

9. (12 pts) Prove the following statement directly from the definition(s).

The product of any two even integers is divisible by 4.