A point $P(x, y, z)$ can be represented as $P(\ \rho,\ \ \theta,\ \ \phi\ )$ in spherical coordinates, where

[Sketch]

$\underline{\quad\rho\quad}$ is the distance between $\underline{\quad P \text{ and the origin} \quad}$ $\Rightarrow$ $\rho^2 = x^2 + y^2 + z^2 \quad \rho \geq 0$

$\underline{\quad\theta\quad}$ measures the angle in the xy-plane from the positive x-axis to the line segment $\overline{OP'}$

Same $\underline{\quad\theta\quad}$ for cylindrical coordinates.

$\underline{\quad\phi\quad}$ measures the angle from the $\underline{\quad \text{positive } z\text{-axis} \quad}$ to the line segment $\underline{\quad\overline{OP}\quad}$. $(\quad 0 \leq \phi \leq \pi \quad)$

To convert between coordinate systems, consider triangle $OP'P$.

[Sketch]

Spherical to Rectangular:

Rectangular to Spherical:

Spherical to Cylindrical:

Cylindrical to Spherical:

<u>EX</u> Given the spherical point $\left(2, \frac{\pi}{4}, \frac{\pi}{3}\right)$

(a). Plot the point

(b). Convert to Rectangular

(c). Convert to Cylindrical

Spherical coordinates are good for describing surfaces that have __symmetry about a point__ .

<u>EX</u>

<u>EX</u> Identify the following surface and sketch. $\rho \sin\theta = 2$

<u>EX</u> Write the equation for a the hyperboloid of one sheet in spherical coordinates. $x^2 + y^2 - z^2 = 1$

Triple Integrals in Spherical Coordinates $\int\int\int_E f(x, y, z)\, dV$

What is the volume element dV?

[Sketch]

$$\int\int\int_E f(x, y, z)\, dV = \int\int\int_E f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)\, dV =$$

where E is the spherical wedge $E = \{(\rho, \theta, \phi) | a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\}$

[Note: Can be extended to more general regions, e.g. $g_1(\theta, \phi) \leq \rho \leq g_2(\theta, \phi)$]

EX Find the volume between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$ and inside the cone $3z^2 = x^2 + y^2$.

EX Sketch the solid region of integration. Convert the integral to spherical coordinates.

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} x \; dz \, dx \, dy$$

EX Convert to spherical and integrate. $\displaystyle\int\int\int_E \frac{\sqrt{x^2 + y^2 + z^2}}{z} \, dV$ where E is the solid bounded by the upper half of the cone $z^2 = x^2 + y^2$ and the plane $z = 1$.