From last time: Maximize $V = xyz$ subject to $2x + 2y + z = 108$.

More Generally: Optimize subject to . (Constraint is ________________)

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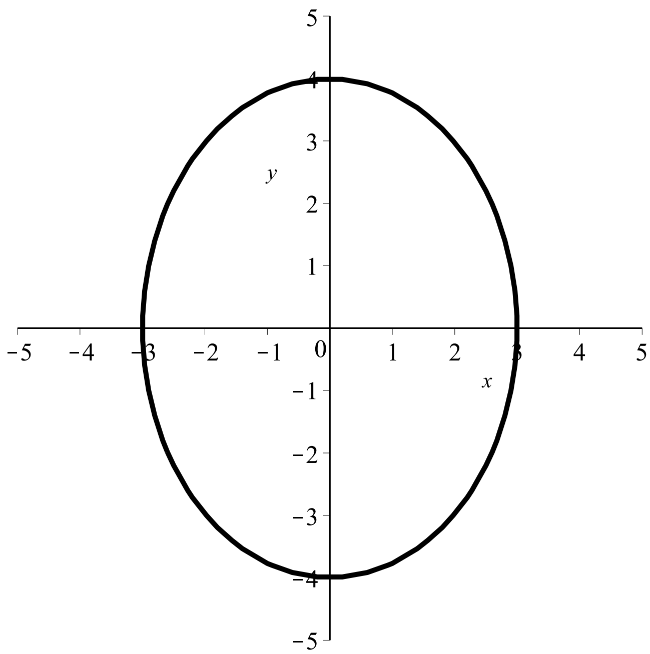
i.e. Find the points (x, y) on the level curve $g(x, y) = C$ that will yield max or min values of f.
[Similar for functions of 3 variables.]

EX Maximize $f(x, y) = 2xy$ for $x, y > 0$ subject to $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$.

The constraint is given in the figure:



Consider level curves of the function $f(x, y) = 2xy$ that we want to maximize. $\Rightarrow$

$k = 4$:

$k = 8$:

$k = 12$:

$k = 16$:

$k = 20$:

What is happening at the point $P_0(x_0, y_0)$ on g where f is maximum?

f and g have a $\Rightarrow$ f and g have the

$\Rightarrow$ Since the direction of the normal line is given by the ______________ , then the gradient vectors at P_0 of f and g are ______________ .

$\Rightarrow$ The gradient vectors are ___________________________________

$\Rightarrow$ OR

for some scalar λ ( ).

Recall, we are trying to find the point (x_0, y_0) [ ] and we just introduced λ [ ].

$\Rightarrow$ [ ]

From (*): $\langle f_x, f_y \rangle = \lambda \langle g_x, g_y \rangle$.

$\Rightarrow$

and

<u>EX</u> Maximize $f(x, y) = 2xy$ for $x, y > 0$ subject to $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$.

$\nabla f = \langle 2y, 2x \rangle$ and $\nabla g = \left\langle \dfrac{2}{9}x, \dfrac{1}{8}y \right\rangle$

$$\langle 2y, 2x \rangle = \lambda \left\langle \frac{2}{9}x, \frac{1}{8}y \right\rangle$$

EX (from last time) Maximize $V = xyz$ subject to $2x + 2y + z = 108$

$\nabla V = \lambda \nabla g$

EX Find the ***absolute*** extrema of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the disk $x^2 + y^2 \leq 16$.

Interior: $f_x = 4x - 4 = 0$ AND $f_y = 6y = 0$

Boundary: Optimize $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ subject to ________________ .

Constrained Max/Min Problem $\Rightarrow$

Corners: None

Compare $f(x, y)$ at the different points where an absolute max/min could possibly occur:

$$
\begin{aligned}
f(x, y) &= 2x^2 + 3y^2 - 4x - 5 \\
f(1, 0) &= \\
f(4, 0) &= 2(4)^2 + 3(0)^2 - 4(4) - 5 = \\
f(-4, 0) &= 2(-4)^2 + 3(0)^2 - 4(-4) - 5 = \\
f(-2, \pm\sqrt{12}) &= 2(-2)^2 + 3(\pm\sqrt{12})^2 - 4(-2) - 5 =
\end{aligned}
$$

Two Constraints: Optimize $f = f(x, y, z)$ subject to $g(x, y, z) = k$ and $h(x, y, z) = c$.

$\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0) + \mu \nabla h(x_0, y_0, z_0)$

$\Rightarrow$

EX The plane $x - y + 3z = 6$ intersects the sphere $x^2 + y^2 + z^2 = 9$ in an ellipse (curve C). Find the highest and lowest points on the ellipse.