Sketching graphs of surfaces in 3D:

A **cylinder** is a surface that consists of

Note about the equations of cylindrical surfaces.

Ex: Parabolic Cylinder

Ex: Circular Cylinder

Ex: Elliptic Cylinder

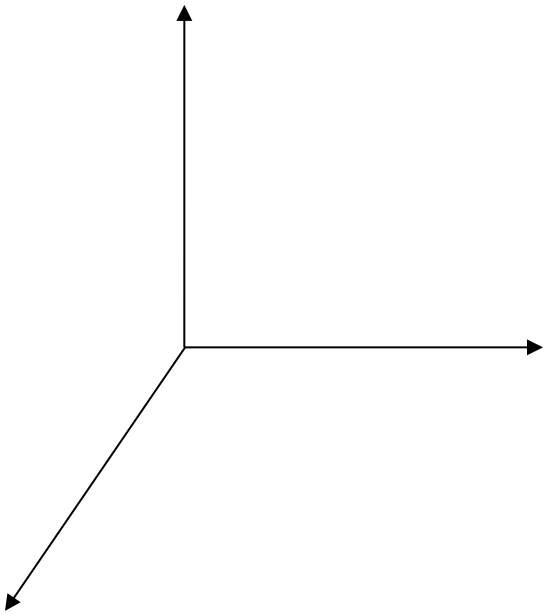
A **quadratic surface** is the graph of a

6 Basic Types

HYPERBOLIC PARABOLOID
ELLIPTIC CONE
HYPERBOLOID OF ONE SHEET
HYPERBOLOID OF TWO SHEETS
ELLIPSOID
ELLIPTIC PARABOLOID

Match the names of the surfaces above to their graph in the examples below.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



xy-plane
z = 0

z = k < 0

z = k > 0

yz-plane
x = 0

x = k < 0

x = k > 0

xz-plane
y = 0

y = k < 0

y = k > 0

HYPERBOLIC PARABOLOID

ELLIPTIC CONE

HYPERBOLOID OF ONE SHEET

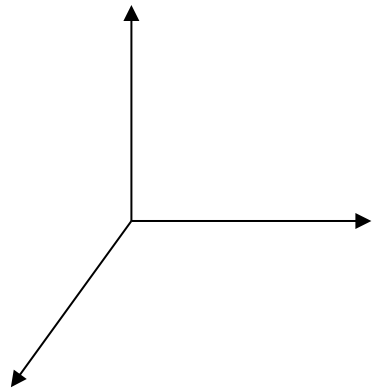
HYPERBOLOID OF TWO SHEETS

ELLIPSOID

ELLIPTIC PARABOLOID

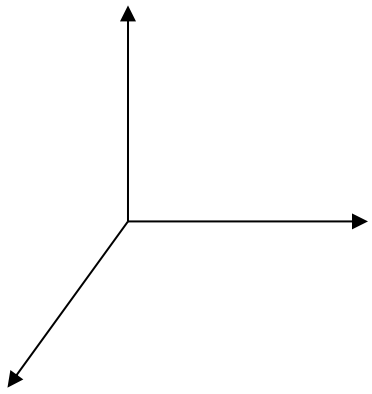
$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

<u>xy-plane</u>
z = 0 ***point***
z = k < 0 ***ellipse***
z = k > 0 ***ellipse***
<u>yz-plane</u>
x = 0 ***2 lines***
x = k < 0 ***hyperbolas***
x = k > 0 ***hyperbolas***
<u>xz-plane</u>
y = 0 ***2 lines***
y = k < 0 ***hyperbolas***
y = k > 0 ***hyperbolas***



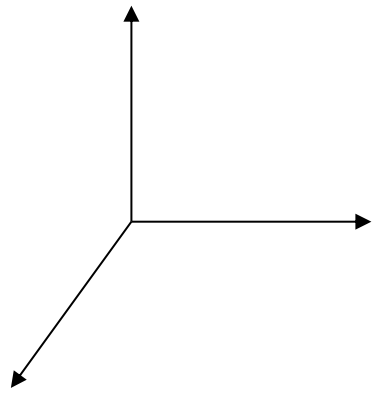
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

<u>xy-plane</u>
z = 0 ***ellipse***
z = k < 0 ***ellipse***
z = k > 0 ***ellipse***
<u>yz-plane</u>
x = 0 ***hyperbola***
x = k < 0 ***hyperbola***
x = k > 0 ***hyperbola***
<u>xz-plane</u>
y = 0 ***hyperbola***
y = k < 0 ***hyperbola***
y = k > 0 ***hyperbola***



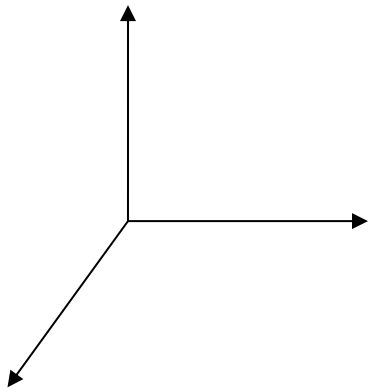
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

<u>xy-plane</u>
z = 0
z = k < 0
z = k > 0
<u>yz-plane</u>
x = 0
x = k < 0
x = k > 0
<u>xz-plane</u>
y = 0
y = k < 0
y = k > 0



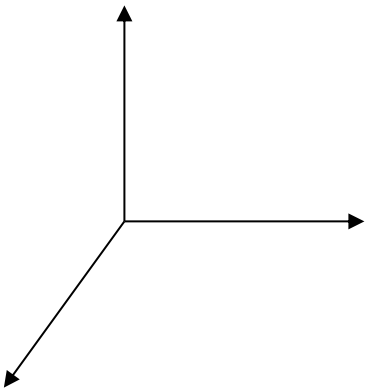
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

<u>xy-plane</u>
z = 0 ***point***
z = k < 0 ***not possible***
z = k > 0 ***ellipse***
<u>yz-plane</u>
x = 0 ***parabola***
x = k < 0 ***parabola***
x = k > 0 ***parabola***
<u>xz-plane</u>
y = 0 ***parabola***
y = k < 0 ***parabola***
y = k > 0 ***parabola***



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$

<u>xy-plane</u>
z = 0
z = k < 0
z = k > 0
<u>yz-plane</u>
x = 0
x = k < 0
x = k > 0
<u>xz-plane</u>
y = 0
y = k < 0
y = k > 0



Ex: Reduce the equation to one of the standard forms, classify the surface, and sketch it.

$4x^2 - 8x - 16y^2 - 96y - z^2 = 124$

Ex: Use traces to sketch and identify the surface $x^2 + 4y^2 + z^2 = 4$.

Ex: Sketch the surface $z = \sin y$.