

1. By looking at the definition, Property: $\mathbf{a} \times \mathbf{a} = \underline{\mathbf{0}}$

2. Given $\mathbf{a} = -3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 3\mathbf{j} + \mathbf{k}$,

(a). Find $\mathbf{a} \times \mathbf{b}$

(b). Find $\mathbf{b} \times \mathbf{a}$

State a property of the cross product that you expect to be true from parts (a) and (b).

Property: $\underline{\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})}$

Using your result of 2(a), find

(c). $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a}$

(d). $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b}$

Write down the two properties from parts (c) and (d).

Property: $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \underline{\mathbf{0}}$ $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \underline{\mathbf{0}}$

Fill in the blanks below for the property implied by the previous property.

Property: The *vector* $\mathbf{a} \times \mathbf{b}$ is orthogonal to both vectors \mathbf{a} and \mathbf{b} since the dot product is $\mathbf{0}$.

3. Property: If θ is the angle between \mathbf{a} and \mathbf{b} , then $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$.

See book for proof.

i.e. The *magnitude* of the cross product equals the magnitude of \mathbf{a} times the magnitude of \mathbf{b} times $\sin \theta$.

(a). Suppose \mathbf{a} and \mathbf{b} are nonzero vectors such that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. What are the possible angles θ between \mathbf{a} and \mathbf{b} ?

Fill in the blank for a property of the cross product that you expect to be true from part (a).

Property: The vectors \mathbf{a} and \mathbf{b} are parallel if and only if $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

4. Recall that the general formula for the area of a parallelogram is $A = \text{base} \cdot \text{height}$.
Given the parallelogram formed from two vectors \mathbf{a} and \mathbf{b} [See board]

(a). What is the length of the base of the parallelogram?

[Don't over think it.]

(b). Write an expression for the height of the parallelogram using the angle θ and magnitudes of the vectors \mathbf{a} and/or \mathbf{b} .

[Think right triangle and trig.]

(c). Using parts (a) & (b), what is the area of the parallelogram ($A = \text{base} \cdot \text{height}$) in terms of the vector magnitudes and angles?

(d). Rewrite the formula for the area of a parallelogram in terms of the cross product.

(e). Find the area of a parallelogram with vertices $P(0, 0, 0)$, $Q(5, 0, 0)$, $R(2, 6, 6)$ and $S(-3, 6, 6)$.