

Quantities

- SCALAR: Measures magnitude only eg. length, speed, mass, etc.
- VECTOR: Measures magnitude and direction eg. displacement, velocity, force, etc.

Graphical Representation of Vectors: line segment and arrow
 (length = magnitude) (direction)

[Sketch Picture with initial point, terminal point, and hand-written notation]

Two vectors are equal iff [Sketch picture both w/ & w/o coord. system. Initial/terminal points don't matter. Graph Rep not unique.]

- (a). Equal magnitude
- (b). Same direction

Mathematical Representation:

2-Dimensional
 Vector Space V_2

$$\mathbf{v} = \vec{v} = \langle a, b \rangle$$

components of \mathbf{v} : a, b

magnitude:

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$

3-Dimensional
 Vector Space V_3

$$\mathbf{v} = \vec{v} = \langle a, b, c \rangle$$

magnitude:

$$|\mathbf{v}| = \sqrt{a^2 + b^2 + c^2}$$

n-Dimensional
 Vector Space V_n

$$\mathbf{v} = \vec{v} = \langle a_1, a_2, a_3, \dots, a_n \rangle$$

magnitude:

$$|\mathbf{v}| = \sqrt{a_1^2 + a_2^2 + a_3^2 + \dots + a_n^2}$$

POSITION VECTOR: The unique vector starting at the origin O and ending at the point $P(a, b)$ or $P(a, b, c)$. [Sketch]

Ex: Sketch $\mathbf{v} = \langle 2, -3 \rangle$ starting at (a). $P(-1, -2)$ (b). Origin

Ex: Find the length of $\vec{w} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$

Ex. Given $P(2, 9)$ and $Q(-1, 6)$ (a). Sketch \vec{PQ} (b). Find \vec{PQ} (c). Find \vec{QP}

General Formula: For $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the vector $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

Scalar Multiplication: $c\mathbf{v} = c \langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$

Ex. Given $\mathbf{v} = \langle 2, 1 \rangle$,

(a). Find and sketch $3\mathbf{v}$

(b). Compare $|\mathbf{v}|$ and $|3\mathbf{v}|$.

[Comments on $|c\mathbf{v}|$ and $c\mathbf{v}$.]

Two vectors \mathbf{v} and \mathbf{w} are parallel iff $\mathbf{w} = c\mathbf{v}$

ie. same or opposite direction