2D Vector Applications: Consider a 2D position vector $\mathbf{v} = \langle a, b \rangle$.

[Sketch picture and label.]

From trigonometry: $\cos\theta = \dfrac{a}{|\mathbf{v}|}$ and $\sin\theta = \dfrac{b}{|\mathbf{v}|}$

$a = |\mathbf{v}|\cos\theta$ and $b = |\mathbf{v}|\sin\theta$ give the horizontal and vertical components of the vector.

<u>Ex</u> Find the vector in component form that has length 2 and direction $\dfrac{\pi}{6}$.

<u>Ex</u> (a). If the wind is blowing at 55 mph in the N 30° E direction, find the wind velocity vector $\mathbf{w}$.

(b). If a jet is flying in still air at 765 mph in the N 45° W direction, find the jet velocity vector $\mathbf{v}$.

(c). Find the true velocity (ground speed) of the jet flying in the 55 mph wind.

Ex: A 200 lb. traffic light is supported by two cables, which make 15° and 20° angles with the horizontal (see figure). The light hangs in equilibrium (all forces balance). Find the forces (tensions) $\mathbf{T}_1$ and $\mathbf{T}_2$ in both cables.

$\mathbf{T}_1 = \langle |\mathbf{T}_1| \cos 165^\circ, |\mathbf{T}_1| \sin 165^\circ \rangle = \langle t_1, t_2 \rangle$

$\mathbf{T}_2 = \langle |\mathbf{T}_2| \cos 20^\circ, |\mathbf{T}_2| \sin 20^\circ \rangle = \langle s_1, s_2 \rangle$

$\mathbf{w} = \langle 0, -200 \rangle$

Forces Balance: $\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{w} = \mathbf{0}$

i.e. $\langle t_1, t_2 \rangle + \langle s_1, s_2 \rangle + \langle 0, -200 \rangle = \langle 0, 0 \rangle \quad \Rightarrow \quad \langle t_1 + s_1, t_2 + s_2 - 200 \rangle = \langle 0, 0 \rangle$

Two vectors are equal if their components are equal. $\Rightarrow$ $t_1 + s_1 = 0$ and $t_2 + s_2 - 200 = 0$

i.e. $t_1 + s_1 = 0$ and $t_2 + s_2 = 200$ Looks like 2 equations, 4 unknowns, but really...

Substitute expressions for $t_1, t_2, s_1,$ and s_2

$t_1 + s_1 = 0 \Rightarrow |\mathbf{T}_1| \cos 165^\circ + |\mathbf{T}_2| \cos 20^\circ = 0 \qquad (1)$

$t_2 + s_2 = 200 \Rightarrow |\mathbf{T}_1| \sin 165^\circ + |\mathbf{T}_2| \sin 20^\circ = 200 \qquad (2)$ only 2 unknowns $|\mathbf{T}_1|$ and $|\mathbf{T}_2|$

Solve for $|\mathbf{T}_2|$ in equation (1): $|\mathbf{T}_2| = \dfrac{-|\mathbf{T}_1| \cos 165^\circ}{\cos 20^\circ}$

Substitute into (2): $|\mathbf{T}_1| \sin 165^\circ + \dfrac{-|\mathbf{T}_1| \cos 165^\circ}{\cos 20^\circ} \sin 20^\circ = 200$

And solve for $|\mathbf{T}_1|$: $|\mathbf{T}_1| \cdot (\sin 165^\circ - \cos 165^\circ \tan 20^\circ) = 200$

$$|\mathbf{T}_1| = \frac{200}{\sin 165^\circ - \cos 165^\circ \tan 20^\circ} \approx 327.66 \text{ lb. force}$$

$$\Rightarrow |\mathbf{T}_2| = \frac{-327.66 \cos 165^\circ}{\cos 20^\circ} \approx 336.81 \text{ lb. force}$$

Finally the tension vectors are:

$\mathbf{T}_1 = \langle 327.66 \cos 165^\circ, 327.66 \sin 165^\circ \rangle \approx \langle -316.50, 84.80 \rangle$

$\mathbf{T}_2 = \langle 336.81 \cos 20^\circ, 336.81 \sin 20^\circ \rangle \approx \langle 316.50, 115.20 \rangle$