Name: Key

Math 251 Calculus III – Crawford

Quiz 3

29 March 2017

Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (4 pts) Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$$

Path 1: $y = mx$ $\Rightarrow$ $\lim_{x\to 0} \frac{x^2(mx)}{x^4+(mx)^2} = \lim_{x\to 0} \frac{mx^3}{x^4+m^2x^2} = \lim_{x\to 0} \frac{mx^3}{x^2(x^2+m^2)}$

$= \lim_{x\to 0} \frac{mx}{x^2+m^2} = \frac{0}{0+m^2} = 0$

Path 2: $y = x^2$ $\Rightarrow$ $\lim_{x\to 0} \frac{x^2(x^2)}{x^4+(x^2)^2} = \lim_{x\to 0} \frac{x^4}{x^4+x^4} = \lim_{x\to 0} \frac{x^4}{2x^4}$

$= \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$

Different limits along different paths $\Rightarrow$ Limit DNE

2. (4 pts) Use polar coordinates to find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$$

$x = r\cos\theta$ $\quad y = r\sin\theta$

$x^2+y^2 = r^2$

$= \lim_{r\to 0} \frac{\sin(r^2)}{r^2} \overset{L}{=} \lim_{r\to 0} \frac{\cos(r^2)\cdot 2r}{2r} = \lim_{r\to 0} \cos(r^2)$

$\frac{0}{0}$

$= \cos(0)$

$= \boxed{1}$

3. (7 pts) Given $f(x,y) = \ln(x^2y + 3x)$.

(a). Find f_x.

$$f_x = \frac{1}{x^2y+3x} \cdot (2xy+3) = \boxed{\frac{2xy+3}{x^2y+3x}}$$

(b). Find f_{xy}.

$$f_{xy} = \boxed{\frac{(x^2y+3x)(2x) - (2xy+3)(x^2)}{(x^2y+3x)^2}}$$

$$= \frac{2x^3y + 6x^2 - 2x^3y - 3x^2}{(x^2y+3x)^2}$$

$$= \frac{3x^2}{(x^2y+3x)^2}$$