Name: Key **Quiz 2**

Math 251 Calculus III – Crawford **01 March 2016**

Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (4 pts) Reduce the following equation to one of the standard forms and classify the surface.
[ellipsoid, cone, elliptic paraboloid, hyperboloid of one sheet, hyperboloid of two sheets, hyperbolic paraboloid].

$x^2 + 4x - y^2 - 4z^2 + 4 = 0$

$$\underbrace{x^2+4x+4}\overbrace{-\cancel{4}-y^2-4z^2+\cancel{4}}=0$$

$$(x+2)^2-y^2-4z^2=0$$

$$\boxed{(x+2)^2=y^2+4z^2 \quad \text{cone}}$$

2. (5 pts) Find the limit, if it exists. If it does not exist, clearly indicate the reason why. [Simplify.]

$$\lim_{t\to 0}\left\langle 3e^{-4t}, \cos(2t), \frac{\sin(5t)}{t}\right\rangle = \boxed{\langle 3, 1, 5\rangle}$$

$$\lim_{t\to 0} 3e^{-4t} = 3e^0 = \boxed{3}$$

$$\lim_{t\to 0} \cos(2t) = \cos(0) = \boxed{1}$$

$$\lim_{t\to 0} \frac{\sin(5t)}{t} = \frac{\sin(0)}{0} = \frac{0}{0}$$

Indeterminate Form

$\Rightarrow$ More Work!!

$$\rightarrow \lim_{t\to 0}\frac{\sin(5t)}{t} \overset{L}{=} \lim_{t\to 0}\frac{5\cos(5t)}{1} = 5\cos(0) = \boxed{5}$$

3. (6 pts) Find a parametric equation for the tangent line to the curve with the given parametric equations at the specified point.

$x = t^2 + 3, \quad y = 2\sqrt{t}, \quad z = \ln(t^2); \qquad (4, 2, 0)$

① pt ✓ (4,2,0) ← occurs when $t = 1$

② direction: $\vec{r}(t) = \langle t^2+3,\ 2\cdot t^{1/2},\ 2\ln t \rangle$

$\vec{r}\,'(t) = \langle 2t,\ t^{-1/2},\ \frac{2}{t} \rangle = \langle 2t,\ \frac{1}{\sqrt{t}},\ \frac{2}{t} \rangle$

$\vec{r}\,'(1) = \langle 2, 1, 2 \rangle$

$t = 1$

$\boxed{x = 4 + 2t, \quad y = 2 + t \quad z = 2t}$ Tangent Line

4. (5 pts) Evaluate the following integral for the *two-dimensional* vector-valued function.

$\int 2t^3\mathbf{i} + \sin(2t)\mathbf{j}\ dt \quad = \frac{2}{4}t^4\ \hat{\imath} - \frac{1}{2}\cos(2t)\ \hat{\jmath} + \vec{C}$

$= \boxed{\frac{1}{2}t^4\ \hat{\imath} - \frac{1}{2}\cos(2t)\hat{\jmath} + \vec{C}}$

(Note: $\vec{C} = C_1\hat{\imath} + C_2\hat{\jmath}$)