1. Find and sketch the domain of $f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$.

2. Draw a contour map for the function $f(x, y) = x^2 - y$. In particular, neatly draw and label the level curves $f(x, y) = k$ for $k = -1, 0, 1, 2$.

3. Find the limit, if it exists, or show that the limit does not exist.

(a). $\lim_{(x,y)\to(0,0)} \frac{xy + 1}{x^2 + y^2 + 1} = 1$ **(b).** $\lim_{(x,y,z)\to(0,0,0)} \frac{x^2 + 2y^2 + 3z^2}{x^2 + y^2 + z^2}$ *DNE*

4. Given $f(x, y) = \begin{cases} \frac{3xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0), \end{cases}$ show that f is not continuous at (0,0).

$f(0, 0) \neq \lim_{(x,y)\to(0,0)} f(x, y)$ since $f(0, 0) = 0$, but $\lim_{(x,y)\to(0,0)} f(x, y)$ DNE

5.

(a). Find the first partial derivatives of $g(x, y) = \frac{x}{x + 2y}$. $g_x = \frac{2y}{(x + 2y)^2}$ $g_y = \frac{-2x}{(x + 2y)^2}$

(b). Find all second partial derivatives of $f(s, t) = \ln(3s^2 - t)^2$ $f_{ss} = \frac{-12(3s^2 + t)}{(3s^2 - t)^2}$ $f_{st} = \frac{12s}{(3s^2 - t)^2}$ $f_{tt} = \frac{-2}{(3s^2 - t)^2}$

(c). Use implicit differentiation to find $\partial z/\partial x$ for $xyz = \cos(xyz)$. $\frac{\partial z}{\partial x} = \frac{-z}{x}$

6. Given the surface $x^2 + y^2 + z^2 - 4y - 2z + 2 = 0$ and the surface represented by the graph of $f(x, y) = \frac{3}{2}x^2 + y^2 - \frac{1}{2}$.

(a). Determine whether the surfaces are tangent, normal, or neither at the point (1,1,2). normal

(b). Find the equation of the tangent plane to the surface $f(x, y) = \frac{3}{2}x^2 + y^2 - \frac{1}{2}$ at the point (1,1,2).

$3x + 2y - z = 3$

7. Find the linear approximation of the function $f(x, y) = e^x \cos xy$ at the point (0,0). $L(x, y) = 1 + x$

8. If $z = y + f(x^2 - y^2)$, where f is differentiable, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$.

Let $t = x^2 - y^2$, then $\frac{\partial z}{\partial x} = 0 + \frac{df}{dt}\frac{\partial t}{\partial x}$ and $\frac{\partial z}{\partial y} = 1 + \frac{df}{dt}\frac{\partial t}{\partial y}$. Then substitute into the above equation.

9. Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ if the equation $\ln(x + yz) = 1 + xy^2 z$ implicitly defines $z = z(x, y)$.

$\frac{\partial z}{\partial x} = \frac{y^2 z - \frac{1}{+yz}}{\frac{y}{x + yz} - xy^2}$ and $\frac{\partial z}{\partial y} = \frac{2xyz - \frac{z}{x + yz}}{\frac{y}{x + yz} - xy^2}$

10. Given $z = z(x, y)$ where $x = 2r + 3s$ and $y = r^2$, find $\dfrac{\partial^2 z}{\partial r^2}$ $\quad \dfrac{\partial^2 z}{\partial r^2} = 4\dfrac{\partial^2 z}{\partial x^2} + 8r\dfrac{\partial^2 z}{\partial y \partial x} + 4r^2\dfrac{\partial^2 z}{\partial y^2} + 2\dfrac{\partial z}{\partial y}$

11. Write out the chain rule for $f = f(x, y, z)$, where $x = x(r, s), y = y(r, s), z = z(r, s)$

$\dfrac{\partial f}{\partial r} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial r} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial r} + \dfrac{\partial f}{\partial z} \cdot \dfrac{\partial z}{\partial r}$ and $\dfrac{\partial f}{\partial s} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial s} + \dfrac{\partial f}{\partial z} \cdot \dfrac{\partial z}{\partial s}$

12. Find the directional derivative of $g(x, y) = \arctan(xy^2)$ at $(2, 1)$ in the direction of $\mathbf{v} = \langle 2, -3 \rangle$ $\quad \dfrac{-2}{\sqrt{13}}$

13. Given $T(x, y, z) = 3x^2 + 4y^2 + 5z$ is the temperature at a point (x, y, z).

(a). How fast (in degrees per unit distance) is the temperature changing at the point P(1,-1,2) in the direction of Q(3,2,-4)? $\quad D_u T = -6$

(b). In which direction does the temperature increase the fastest at P? $\quad \langle 6, -8, 5 \rangle$

14. Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line that joins the points $(5, 3, 6)$ and $(8, 4, 10)$. $\quad (3/4, -1/4, 1/2)$ and $(-3/4, 1/4, -1/2)$

15. Given $f(x, y) = 1 - x^3 + 4xy - 2y^2$

(a). Find all the critical points of f. Determine if each critical point yields a relative maximum or minimum or a saddle point. $\quad (0, 0)$ is saddle; $(4/3, 4/3)$ is relative maximum

(b). Find the absolute maximum and minimum values of f on the set D defined as the closed triangular region in the xy-plane with vertices (0,0), (0,12), and (12,0).

Absolute max value = 1 at pt.(0,0). Absolute min value = −1727 at pt. (12,0)

16. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin. [Use both the "regular" method (Section 14.7) and the method of Lagrange Multipliers (Section 14.8).] $\quad \left(\frac{1}{\sqrt[4]{3}}, \pm\frac{\sqrt{2}}{\sqrt[4]{3}}, \sqrt[4]{3}\right)$ and $\left(-\frac{1}{\sqrt[4]{3}}, \pm\frac{\sqrt{2}}{\sqrt[4]{3}}, -\sqrt[4]{3}\right)$

17. Use the method of Lagrange Multipliers to find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$. $\quad (2, 1, \sqrt{5})$ and $(2, 1, -\sqrt{5})$

18. Sketch the solid whose volume is given by the iterated integral $\displaystyle\int_0^1 \int_0^1 2 - x^2 - y^2 \, dy \, dx$.

19. Evaluate the following integrals

(a). $\displaystyle\int_0^1 \int_0^x \cos(x^2) \, dydx \quad = \frac{1}{2}\sin 1$

(b). $\displaystyle\iint_D y \, dA$ where D is the region in the first quadrant that lies above $y = \dfrac{1}{x}$ and $y = x$ and below the line $y = 2$. $\quad \dfrac{4}{3}$

20. Rewrite the integral using the indicated order of integration. (**Do NOT evaluate**):

(a). $\displaystyle\int_{-2}^{2}\int_{y^2}^{4} y\,dx\,dy$ Rewrite using $dy\,dx$ $\displaystyle\int_0^4\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx$

(b). $\displaystyle\int_0^2\int_{x^2}^{x+2} dy\,dx$ Rewrite using $dx\,dy$ $\displaystyle\int_0^2\int_0^{\sqrt{y}} dxdy+\int_2^4\int_{y-2}^{\sqrt{y}} dxdy$

21. Find the volume of the solid region in the 1st Octant bounded by the coordinate planes and the plane $2x + 3y + 4z = 12$. Volume $= 12$

22. Use a double integral and polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the the sphere $x^2 + y^2 + z^2 = 1$. (**Set up the integral, but do NOT evaluate.**)

$\displaystyle\iint_D \sqrt{1-x^2-y^2}-\sqrt{x^2+y^2}\,dA$ where $D = x^2+y^2 = 1/2 \Longrightarrow \displaystyle\int_0^{2\pi}\int_0^{1/\sqrt{2}}(\sqrt{1-r^2}-\sqrt{r^2})r\,drd\theta = \int_0^{2\pi}\int_0^{1/\sqrt{2}} r\sqrt{1-r^2}-r^2\,drd\theta$