**Name:** ________________________________ **Exam 2**

**Math 251 Multivariate Calculus – Crawford** **26 April 2017**

- You may use the provided calculator and formula sheet. But show all intermediate steps and ***leave all answers exact***.
- Other notes (in any form), books, phones, and any other unauthorized sources are not allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Unless otherwise stated, simplify/evaluate trigonometric, exponential, logarithmic, and hyperbolic functions for standard values.
- Good Luck!

Calculator Number: __________

Score

| | |
|---|---|
| 1 | /6 |
| 2 | /10 |
| 3 | /14 |
| 4 | /14 |
| 5 | /14 |
| 6 | /18 |
| 7 | /10 |
| 8 | /8 |
| 9 | /8 |
| 10 | /2 |
| Total | /100 |

**1.** (6 pts). Find and sketch the domain of $f(x,y) = \dfrac{\sqrt{x}}{4 - x^2 - y^2}$

**2.** (10 pts). Find the limit, if it exists, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

**3.** (14 pts). For each of the given functions, find the indicated partial derivative. [You do not need to simplify.]

**(a)**. Find $f_{yyx}$ for $f(x,y) = xy^4e^{-x}$.

**(b)**. Find $\dfrac{\partial z}{\partial y}$ where $z = z(x,y)$ is defined implicitly by $yz + x\ln y = z^2$.

**4.** (14 pts). Given $f(x, y) = \dfrac{x}{y^2}$,

**(a)**. Find an equation of the tangent plane to the surface $z = f(x, y)$ at the point $(-4, 2)$.

**(b)**. Find the linearization $L(x, y)$ of $f$ at the point $(-4, 2)$.

**(c)**. Use your answer from part (b) to approximate $f(-3.9, 2.2)$.

**5.** (14 pts). Suppose that over a certain region of space the electrical potential $V$ is given by

$V(x, y, z) = 6x^2 - 4xy + 2xyz,$

**(a)**. Find the rate of change of the potential $V$ at $P(-2, -1, 3)$ in the direction of $\langle 1, 3, -1 \rangle$.

**(b)**. In which direction is $V$ decreasing the fastest at $P(-2, -1, 3)$?

**6.** (18 pts). Find all the critical points of $f$ and determine if each yields a local maximum or minimum or is a saddle point. In the case of a maximum or minimum, find the local maximum or minimum value(s).

$f(x, y) = x^3 - 3x + \dfrac{1}{3}xy^2,$

**7.** (10 pts). Evaluate the following integral. [Show all work. Leave your answer exact and simplify.]

$$\int_0^{\sqrt[3]{\pi}} \int_0^{x^2} y^2 + \sin(x^3)\; dy\; dx$$

**8.** (8 pts). Sketch the region of integration and change the order. [Do not evaluate.]

$$\int_0^4 \int_{\sqrt{x}}^2 x^2 y\; dy\; dx$$

**9.** (8 pts). Convert the following integral to polar coordinates. [Do not evaluate.]

$$\int_0^3 \int_{-\sqrt{9-x^2}}^0 \frac{y^2}{x^2+y^2}\, dy\, dx$$

**10.** (2 pts). Multiple Choice

Given that $f = f(x, y, z)$ where $x = x(s, t), y = y(s, t)$, and $z = z(s, t)$, circle the correct form of the chain rule.

**(a).** $\dfrac{\partial f}{\partial s} = \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial x} + \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial y} + \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial z}, \qquad \dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial x} + \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial y} + \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial z}$

**(b).** $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial x} + \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial x}, \qquad \dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial y} + \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial y}, \qquad \dfrac{\partial f}{\partial z} = \dfrac{\partial f}{\partial s} \cdot \dfrac{\partial s}{\partial z} + \dfrac{\partial f}{\partial t} \cdot \dfrac{\partial t}{\partial z}$

**(c).** $\dfrac{\partial f}{\partial s} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial s} + \dfrac{\partial f}{\partial z} \cdot \dfrac{\partial z}{\partial s}, \qquad \dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial t} + \dfrac{\partial f}{\partial z} \cdot \dfrac{\partial z}{\partial t}$