1. Given the vector $\mathbf{v} = \langle 2, 3, -4 \rangle$

(a). Find a unit vector in the same direction as $\mathbf{v}$ $\mathbf{u} = \left\langle 2/\sqrt{29}, 3/\sqrt{29}, -4/\sqrt{29} \right\rangle$

(b). Find a vector in the opposite direction as $\mathbf{v}$ that has a magnitude of 3. $\mathbf{u} = \left\langle -6/\sqrt{29}, -9/\sqrt{29}, 12/\sqrt{29} \right\rangle$

2. Determine whether the the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ are orthogonal, parallel, or neither. neither

3. Show that the following equation represents a sphere and find its center and radius.
$x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$ $r = 5$; Center $(1, -3, -4)$

4. Given the points $P(1, 0, 1), Q(1, -1, 4)$, and $R(3, -1, 1)$,

(a). Find a vector orthogonal to the plane through the points P, Q, and R. $\langle 3, 6, 2 \rangle$

(b). Find the area of the triangle PQR. 7/2

5. A constant force is given by the vector $F = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and moves an object along a straight line from the points P(2,1,0) and Q(3,5,2). Find the work done. 29

6. Describe in your own words the vector and scalar projections of $\mathbf{b}$ onto $\mathbf{a}$. Use a sketch if necessary.

7. Given $\mathbf{u} = \langle 1, 2, 4 \rangle$, $\mathbf{v} = \langle 2, -1, 2 \rangle$, and $\mathbf{w} = \langle 1, 0, -1 \rangle$,

(a). Are the three vectors coplanar? No **(b).** If not, find the scalar and vector projections of $\mathbf{u}$ onto $\mathbf{w}$. $\left\langle -\frac{3}{2}, 0, \frac{3}{2} \right\rangle$

8. Given the point $(2, 2, 1)$ and the line $x = 2t, y = 4 + 2t, z = t$,

(a). Find the equation of a plane that passes through the point and contains the line. $-4x + 8z = 0$

(b). Find the symmetric equations of a line passing through $(2, 2, 1)$ and perpendicular to the plane found in part (a).
$\frac{x-2}{-4} = \frac{z-1}{8}; y = 2$

9. Given the two planes $5x - 3y + z = 4$ and $x + 4y + 7z = 1$

(a). Determine whether they are parallel, perpendicular, or neither. perpendicular

(b). If they are parallel, find the distance between them. If they are neither, find the cosine of the angle between them.
N/A

10. Reduce the equation $4x^2 - y^2 - 2z^2 + 12z = 18$ to one of the standard forms, classify the surface, and describe (or sketch) it. Cone with axis parallel to the x-axis and shifted up 3 units.

11. Section 12.6, p. 857 #21-28 (matching) 21) VII 22) IV 23) II 24) III 25) VI 26) I 27) VIII 28) V

12. Find the domain of the vector function $\left\langle \frac{t}{3t-1}, e^{1/t}, \cos(t) \right\rangle$ All real numbers except $t \neq \frac{1}{3}, 0$

13. Evaluate the following limits.

(a). $\lim_{t\to\infty} \left\langle e^{-t}, \frac{1}{t}, \frac{t^2}{t^2+1} \right\rangle = \langle 0, 0, 1 \rangle$

(b). $\lim_{t\to 2} \left(t\mathbf{i} + \frac{t^2-4}{t^2-2t}\mathbf{j} + \frac{1}{t}\mathbf{k} \right) = 2\mathbf{i} + 2\mathbf{j} + \frac{1}{2}\mathbf{k}$

14. Section 13.1, p. 870: #21-26 (matching) 21. II 22. VI 23. V 24. I 25. IV 26.III

15. Sketch the plane curve given by the vector equation $\mathbf{r}(t) = 3\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}$. Indicate with an arrow the direction in which t increases. Sketch a parabola $z = y^2$ in the plane $x = 3$. It is traversed right to left.

16. Find a vector function that represents the curve of intersection of the paraboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.
$\mathbf{r}(t) = \langle \cos t, \sin t, \cos 2t \rangle$

17. Given the space curve with the parametric representation $x = 9\cos t, y = 9\sin t$, and $z = e^t$,

(a). Sketch the curve and indicate with an arrow the direction of increasing t. The curve spirals along the surface of the cylinder $x^2 + y^2 = 81$. The spirals are close and tight near the plane $z = 0$ and grow exponentially.

(b). Find the tangent vector when $t = \dfrac{\pi}{2}$ and sketch it on the curve. $\mathbf{r}'(t) = < -9, 0, e^{\pi/2} >$

18. Evaluate the following integrals:

(a). $\displaystyle\int_1^e \frac{1}{t}\mathbf{i} + t\mathbf{j} + \frac{1}{t^2}\mathbf{k}\, dt = \mathbf{i} + \left(\frac{1}{2}e^2 - \frac{1}{2}\right)\mathbf{j} + \left(-\frac{1}{e} + 1\right)\mathbf{k}$

(b). $\displaystyle\int \frac{t}{1+t^2}\mathbf{i} + \cos 2t\mathbf{j} + \sqrt{t}\mathbf{k}\, dt = \frac{1}{2}\ln\left|1 + t^2\right|\mathbf{i} + \frac{1}{2}\sin 2t\mathbf{j} + \frac{2}{3}t^{3/2}\mathbf{k} + \mathbf{C}$

19. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\mathbf{i} - e^{2t}\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. $\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\mathbf{i} + \left(-\frac{1}{2}e^{2t} + \frac{5}{2}\right)\mathbf{j} + 3\mathbf{k}$

20. Given the curve represented by position vector $\mathbf{r}(t) = \langle 5t^2 - t, 3t, \sin \pi t \rangle$

(a). Find the unit tangent vector at $P(4, 3, 0)$. $T(1) = \dfrac{1}{\sqrt{90 + \pi^2}}\langle 9, 3, -\pi \rangle$

(b). Find the equation of the tangent line to the curve at the same point $P(4, 3, 0)$. $x = 4 + 9t, y = 3 + 3t, z = -\pi t$

21. Given the position vector $\mathbf{r}(t) = \langle \cos 2t, 0, \sin 2t \rangle$,

(a). Find the length of the curve for $\pi/2 \le t \le \pi$. π

(b). Find the unit tangent, normal, and binormal vectors $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$.
$\mathbf{T} = \langle -\sin 2t, 0, \cos 2t \rangle$; $\mathbf{N} = \langle -\cos 2t, 0, -\sin 2t \rangle$; $\mathbf{B} = \langle 0, -1, 0 \rangle$

(c). Use the formula $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ to find the curvature. $\kappa(t) = 1$ for all values of t.

22. Given the vector equation $\mathbf{r}(t) = \langle 1 + 6t, 3 - 2t, 3t \rangle$ for a curve

(a). Find the arc length function measured from $P(1, 3, 0)$ [i.e. t=0] in the direction of increasing t. $s(t) = 7t$

23. The acceleration of a particle is given by the vector $\mathbf{a}(t) = \mathbf{k}$. Find the velocity vector $\mathbf{v}(t)$ and position vector $\mathbf{r}(t)$ if the initial velocity and position vectors are $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$, $\mathbf{r}(0) = \mathbf{j}$
$\mathbf{v}(t) = \mathbf{i} - \mathbf{j} + t\mathbf{k}$ and $\mathbf{r}(t) = t\mathbf{i} + (1-t)\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$

24. Section 13.4 #28. $x = 400$ when $t \approx 5.41$. Since $y = \approx 11.2$ ft then, it clears the fence. $\Rightarrow$ Home Run.