Name: Key

Math 251, Multivariate Calculus – Crawford

Exam 1

08 March 2017

Score	
1	/12
2	/12
3	/12
4	/12
5	/28
6	/12
7	/6
8	/10
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the given formula sheet.
- Evaluate trigonometric, exponential, logarithmic, etc., functions at standard values.
- ***Show all your work.*** Partial credit may be given for written work.
- Good Luck!

1. (12 pts). Given $\mathbf{a} = \langle 2, -2, 1\rangle$ and $\mathbf{b} = \langle 4, -3, 0\rangle$,

(a). Find $|\mathbf{b} - 3\mathbf{a}|$.

$$\vec{b} - 3\vec{a} = \langle 4, -3, 0\rangle - 3\langle 2, -2, 1\rangle = \langle 4-6, -3+6, 0-3\rangle$$
$$= \langle -2, 3, -3\rangle$$

$$|\vec{b} - 3\vec{a}| = \sqrt{(-2)^2 + (3)^2 + (-3)^2} = \sqrt{4+9+9} = \boxed{\sqrt{22}}$$

(b). Find a vector of length 4 in the opposite direction of $\mathbf{b}$.

$$\vec{u} = \frac{\vec{b}}{|\vec{b}|} = \frac{\langle 4, -3, 0\rangle}{\sqrt{(4)^2 + (-3)^2 + (0)^2}} = \frac{\langle 4, -3, 0\rangle}{\sqrt{16+9}} = \frac{\langle 4, -3, 0\rangle}{\sqrt{25}}$$

$$= \frac{\langle 4, -3, 0\rangle}{5} = \left\langle \frac{4}{5}, \frac{-3}{5}, 0\right\rangle$$

So $-4\vec{u} = \boxed{\left\langle \frac{-16}{5}, \frac{12}{5}, 0\right\rangle}$

2. (12 pts). Given the line $x = t - 1$, $y = 1 + 2t$ and $z = 3 - t$ and the plane $3x - y + 2z = 5$

Subs into

(a). Find the point at which the line intersects the plane.

$$3(t-1) - (1+2t) + 2(3-t) = 5$$

$$3t - 3 - 1 - 2t + 6 - 2t = 5$$

$$-t + 2 = 5$$

$$-t = 3$$

$$t = -3$$

$$x = -3 - 1 = -4$$

$$y = 1 + 2(-3) = -5$$

$$z = 3 - (-3) = 6$$

$$\boxed{(-4, -5, 6)}$$

(b). Find the symmetric equations of a line through the point found in part (a) and parallel to the line given by $\frac{x-3}{4} = y, \quad z = 6.$

① pt $(-4, -5, 6)$

② direction: $\vec{a} = \langle 4, 1, 0 \rangle$

$$\boxed{\frac{x+4}{4} = \frac{y+5}{1}, \quad z = 6}$$

$\frac{z-6}{0}$ undef.

3. (12 pts). Find the equation of the plane through the points $(0,-1,3)$ and and $(-1,-1,0)$ and is perpendicular to the plane $2x+4y-5z=2$.

P Q

Plane ⊥ w/ $\vec{n}_1$

⊥ So its normal vector is parallel to desired plane

① pt. ✓

② $\vec{n}$: $\vec{PQ} = \langle -1-0, -1+1, 0-3 \rangle = \langle -1, 0, -3 \rangle$

$\vec{n}_1 = \langle 2, 4, -5 \rangle$

$$\vec{n} = \vec{PQ} \times \vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

[Do not simplify.]

$= \langle 0(-5)-(-3)(4), (-3)(2)-(-1)(-5), (-1)(4)-(0)(2) \rangle$

$\vec{n} = \langle 12, -11, -4 \rangle \Rightarrow$ Plane: $\boxed{12(x-0)-11(y+1)-4(z-3)=0}$

using $P(0,-1,3)$

4. (12 pts). Evaluate the limit. If the limit does not exist, clearly explain why.

$$\lim_{t\to 0}\left\langle \sqrt{3-t}, \ln(3t+1), \frac{e^{5t}-1}{3t}\right\rangle = \boxed{\left\langle \sqrt{3}, 0, \frac{5}{3}\right\rangle}$$

$$\lim_{t\to 0}\sqrt{3-t} = \sqrt{3}$$

$$\lim_{t\to 0}\ln(3t+1) = \ln 1 = 0$$

$$\lim_{t\to 0}\frac{e^{5t}-1}{3t} \overset{L}{=} \lim_{t\to 0}\frac{5e^{5t}}{3} = \frac{5e^0}{3} = \frac{5}{3}$$

$\frac{e^0-1}{0} = \frac{0}{0}$ Ind. Form

$\Rightarrow$ More Work

5. (28 pts). Given the curve $\mathbf{r}_1(t) = \langle \sin(3t), 2e^{2t}, 1 \rangle$

(a). Find a parametric equation for the tangent line to the curve $\mathbf{r}_1(t)$ at the point $(0, 2, 1)$.

① pt $(0,2,1)$ Note: occurs when $t=0$

② direction: $\vec{r}_1'(t) = \langle 3\cos 3t, 4e^{2t}, 0 \rangle$

$\vec{r}_1'(0) = \langle 3\cos(0), 4e^0, 0 \rangle = \langle 3, 4, 0 \rangle = \vec{a}$

$$\boxed{\begin{aligned} x &= 0 + 3t \\ y &= 2 + 4t \\ z &= 1 \end{aligned}}$$

Occurs when $S = 1$

(b). The curve $\mathbf{r}_1(t)$ intersects with $\mathbf{r}_2(s) = \langle \ln s^3, s^2 + 1, s \rangle$ at the point $(0, 2, 1)$. Find the angle of intersection of the two curves at that point. *[Leave your answer in terms of an inverse trigonometric function.]*

angle between tangent vectors.

$\vec{r}_2(t) = \langle 3\ln s, s^2 + 1, s \rangle$

$\vec{r}_2'(t) = \langle \frac{3}{s}, 2s, 1 \rangle$

$\vec{r}_2'(1) = \langle 3, 2, 1 \rangle = \vec{b}$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$

$\cos\theta = \dfrac{(3)(3) + 4(2) + 0(1)}{\sqrt{3^2+4^2+0^2}\,\sqrt{3^2+2^2+1^2}} = \dfrac{17}{\sqrt{25}\,\sqrt{14}}$

$\cos\theta = \dfrac{17}{5\sqrt{14}} \Rightarrow \boxed{\theta = \cos^{-1}\left(\dfrac{17}{5\sqrt{14}}\right)}$

(c). Find the curvature of $r_1(t)$ at the point $(0, 2, 1)$ using $\kappa = \dfrac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}$. [Do not simplify.]

$\vec{r}_1''(t) = \langle -9\sin 3t, 8e^{2t}, 0 \rangle$ (See (a) for $\vec{r}_1'(t)$ & $\vec{r}_1'(0)$)

$\vec{r}_1''(0) = \langle 0, 8, 0 \rangle$

$r_1'(0) \times r''(0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 0 & 8 & 0 \end{vmatrix} = \langle 0-0, 0-0, 3\cdot 8 - 0 \rangle$

$= \langle 0, 0, 24 \rangle$

$\Rightarrow |r'(0) \times r''(0)| = 24$

$\underbrace{|r'(0)|^3}_{\text{from part (b)}} = (5)^3 = 125 \Rightarrow \boxed{\kappa = \dfrac{24}{125}}$

6. (12 pts). Given $\mathbf{r}'(t) = \sqrt{t}\mathbf{i} + 2t\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j}$, find $\mathbf{r}(t)$. [Write your final answer as a single vector $\mathbf{r}(t)$.]

$\vec{r}'(t) = t^{1/2}\hat{\imath} + 2t\hat{\jmath} + 3\hat{k}$

$\vec{r}(t) = \left(\frac{2}{3}t^{3/2} + C_1\right)\hat{\imath} + \left(t^2 + C_2\right)\hat{\jmath} + \left(3t + C_3\right)\hat{k}$

$\vec{r}(1) = \left(\frac{2}{3} + C_1\right)\hat{\imath} + (1 + C_2)\hat{\jmath} + (3 + C_3)\hat{k} = 2\hat{\imath} + \hat{\jmath}$

$\Rightarrow \quad \frac{2}{3} + C_1 = 2 \qquad 1 + C_2 = 1 \qquad 3 + C_3 = 0$

$C_1 = 2 - \frac{2}{3} = \frac{4}{3} \qquad C_2 = 0 \qquad C_3 = -3$

$$\boxed{\vec{r}(t) = \left(\frac{2}{3}t^{3/2} + \frac{4}{3}\right)\hat{\imath} + t^2\hat{\jmath} + (3t - 3)\hat{k}}$$

7. (6 pts). *True or False.* Clearly indicate whether the following statements are true or false.

T F $\text{proj}_{\mathbf{a}}\mathbf{b} = \mathbf{0}$ only if $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$.

$\frac{\vec{a}\cdot\vec{b}}{|a|^2}\vec{a} = 0$ if $\underbrace{\vec{a}\cdot\vec{b} = 0}_{\text{orthog vectors.}}$

T F The space curve given by $\mathbf{r}(t) = \langle e^t, e^{2t}, \cos t\rangle$ lies on the surface of a parabolic cylinder.

$x = e^t \qquad y = e^{2t} = (e^t)^2 = x^2$

$y = x^2$ Yes.

T F The arc length function is given by $\int_a^b |\mathbf{r}'(t)|\, dt$. ← arc length ↳ just a value

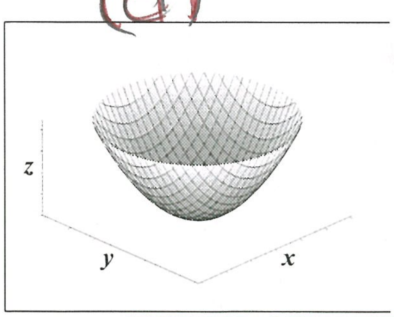
$S(t) = \int_a^t |r'(u)|\,du$

8. (10 pts). Match each of the following graphs with its equation. Also, state the name of each graph.

(a). $y = x^2 - z^2$ (b). $z^2 - y^2 + 2x^2 = 3$ (c). $-z^2 + y^2 - 2x^2 = 3$

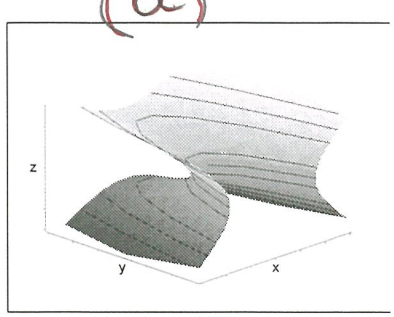
(d). $z = x^2 + y^2$ (e). $x^2 + z^2 = y^2$

(d)



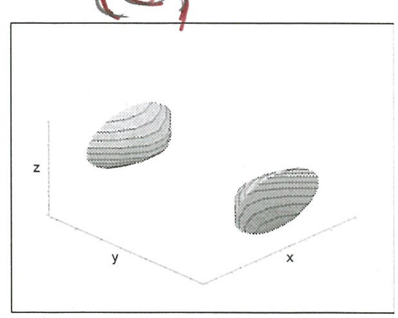
Elliptic Paraboloid

(a)



Hyperbolic Paraboloid

(c)



Hyperboloid of 2 Sheets.