Name: _____ **Exam 1**

Math 251, Multivariate Calculus – Crawford **08 March 2017**

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the given formula sheet.
- Evaluate trigonometric, exponential, logarithmic, etc., functions at standard values.
- ***Show all your work***. Partial credit may be given for written work.
- Good Luck!

	Score
1	/12
2	/12
3	/12
4	/12
5	/28
6	/12
7	/6
8	/10
Total	/100

1. (12 pts). Given $\mathbf{a} = \langle 2, -2, 1\rangle$ and $\mathbf{b} = \langle 4, -3, 0\rangle$,

(a). Find $|\mathbf{b} - 3\mathbf{a}|$.

(b). Find a vector of length 4 in the opposite direction of $\mathbf{b}$.

2. (12 pts). Given the line $x = t - 1$, $y = 1 + 2t$ and $z = 3 - t$ and the plane $3x - y + 2z = 5$

(a). Find the point at which the line intersects the plane.

(b). Find the symmetric equations of a line through the point found in part (a) and parallel to the line given by $\dfrac{x-3}{4} = y, \;\; z = 6.$

3. (12 pts). Find the equation of the plane through the points $(0, -1, 3)$ and and $(-1, -1, 0)$ and is perpendicular to the plane $2x + 4y - 5z = 2$.

[Do not simplify.]

4. (12 pts). Evaluate the limit. If the limit does not exist, clearly explain why.

$$\lim_{t \to 0} \left\langle \sqrt{3 - t}, \ln(3t + 1), \frac{e^{5t} - 1}{3t} \right\rangle$$

5. (28 pts). Given the curve $\mathbf{r}_1(t) = \langle \sin(3t), 2e^{2t}, 1 \rangle$

(a). Find a parametric equation for the tangent line to the curve $\mathbf{r}_1(t)$ at the point $(0, 2, 1)$.

(b). The curve $\mathbf{r}_1(t)$ intersects with $\mathbf{r}_2(s) = \langle \ln s^3, s^2 + 1, s \rangle$ at the point $(0, 2, 1)$. Find the angle of intersection of the two curves at that point. ***[Leave your answer in terms of an inverse trigonometric function.]***

(c). Find the curvature of $r_1(t)$ at the point $(0, 2, 1)$ using $\kappa = \dfrac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}$. [Do not simplify.]

6. (12 pts). Given $\mathbf{r}'(t) = \sqrt{t}\mathbf{i} + 2t\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j}$, find $\mathbf{r}(t)$. [Write your final answer as a single vector $\mathbf{r}(t)$.]

7. (6 pts). *True or False.* Clearly indicate whether the following statements are true or false.

T F $\text{proj}_{\mathbf{a}}\mathbf{b} = \mathbf{0}$ only if $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$.

T F The space curve given by $\mathbf{r}(t) = \langle e^t, e^{2t}, \cos t \rangle$ lies on the surface of a parabolic cylinder.

T F The arc length function is given by $\displaystyle\int_a^b \left|\mathbf{r}'(t)\right| \, dt$.

8. (10 pts). Match each of the following graphs with its equation. Also, state the name of each graph.

(a). $y = x^2 - z^2$ **(b).** $z^2 - y^2 + 2x^2 = 3$ **(c).** $-z^2 + y^2 - 2x^2 = 3$

(d). $z = x^2 + y^2$ **(e).** $x^2 + z^2 = y^2$