Steps for Finding Absolute Extrema of a function $f = f(x, y)$ on a closed and bounded region D.

1. Find the critical points in the interior region.

ie. $f_x = 0$ and $f_y = 0$ OR either DNE

2. Find the critical points on the boundary.

ie.. Reduce it to a one dimensional problem and find associated critical numbers.

3. Find any corners or endpoints of the region D.

4. Evaluate f at all the points found in steps 1-3.

$\Rightarrow$ Max = largest value $\Rightarrow$ Min = smallest value

Ex Find the absolute maximum and minimum values of

$f(x, y) = 2\sin x + 3\cos y$ on the region $D = \{(x, y) | 0 \leq x \leq 2\pi, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\}$

1. Interior:

2. Boundary consists of 4 lines.

L_1:

L_2:

$L_3 : y = \frac{\pi}{2} \Rightarrow g(x) = f(x, \frac{\pi}{2}) = 2\sin x + 3\cos(\frac{\pi}{2}) = 2\sin x$ ie. $g(x) = 2\sin x,\ 0 \le x \le 2\pi$

$g'(x) = 2\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow$ pts: $\boxed{(\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{\pi}{2})}$

$L_4 : x = 0 \Rightarrow g(y) = f(0, y) = 2\sin 0 + 3\cos y = 3\cos y$ ie. $g(y) = 3\cos y,\ \frac{\pi}{2} \le y \le \frac{\pi}{2}$

$g'(x) = -3\sin y = 0 \Rightarrow y = 0 \Rightarrow$ pts: $\boxed{(0, 0)}$

3. Corners: pts $\boxed{(0, -\frac{\pi}{2}), (2\pi, -\frac{\pi}{2}), (2\pi, \frac{\pi}{2}), (0, \frac{\pi}{2})}$

4. Evaluate

Interior:

$f(\frac{\pi}{2}, 0) = 2\sin(\frac{\pi}{2}) + 3\cos 0 = 2 + 3 = 5$

$f(\frac{3\pi}{2}, 0) = 2\sin(\frac{3\pi}{2}) + 3\cos 0 = -2 + 3 = 1$

Boundary (Lines):

$f(\frac{\pi}{2}, -\frac{\pi}{2}) = 2\sin(\frac{\pi}{2}) + 3\cos(-\frac{\pi}{2}) = 2 + 0 = 2$

$f(\frac{3\pi}{2}, -\frac{\pi}{2}) = 2\sin(\frac{3\pi}{2}) + 3\cos(-\frac{\pi}{2}) = -2 + 0 = -2$

$f(2\pi, 0) = 2\sin(2\pi) + 3\cos(0) = 0 + 1 = 1$

$f(\frac{\pi}{2}, \frac{\pi}{2}) = 2\sin(\frac{\pi}{2}) + 3\cos(\frac{\pi}{2}) = 2 + 0 = 2$

$f(\frac{3\pi}{2}, \frac{\pi}{2}) = 2\sin(\frac{3\pi}{2}) + 3\cos(\frac{\pi}{2}) = -2 + 0 = -2$

$f(0, 0) = 2\sin(0) + 3\cos(0) = 0 + 1 = 1$

Boundary (Corners):

$f(0, -\frac{\pi}{2}) = 2\sin(0) + 3\cos(-\frac{\pi}{2}) = 0 + 0 = 0$

$f(2\pi, -\frac{\pi}{2}) = 2\sin(2\pi) + 3\cos(-\frac{\pi}{2}) = 0 + 0 = 0$

$f(2\pi, \frac{\pi}{2}) = 2\sin(2\pi) + 3\cos(\frac{\pi}{2}) = 0 + 0 = 0$

$f(0, \frac{\pi}{2}) = 2\sin(0) + 3\cos(\frac{\pi}{2}) = 0 + 0 = 0$