1. Suppose we just stopped at the on ramp (with a light) to I-290. The following data gives the total distance traveled after we start moving again.

time (sec)	0	1	2	3	4	5
distance (ft)	0	8	32	73	130	203

Calculus Question : Find the *instantaneous* velocity at exactly $t = 1$ second.

But since we don't know how to answer this yet, we're going to approximate it using techniques we already know. $\Rightarrow$ Use *average* velocity.

(a). What is the average velocity over the interval $1 \le t \le 5$ seconds? Note:

$v_{avg} =$

(b). Complete the table below by computing the average velocity for shorter time intervals.

time interval	Δt (sec)	average velocity (ft/sec)
$1 \le t \le 5$	4	48.75
$1 \le t \le 4$	3	
$1 \le t \le 3$	2	
$1 \le t \le 2$	1	

(c). Which velocity do you think is the best estimate of your speed at the exact instant when $t = 1$? Why?

(d). Suppose we have more detailed data in tenths of seconds:

time (sec)	1.1	1.2	1.3	1.4	1.5
distance (ft)	9.81	11.66	13.68	15.91	18.55

Complete the table below by computing the average velocity for even shorter time intervals.

time interval	Δt (sec)	average velocity (ft/sec)
$1 \le t \le 1.5$	0.5	21.10
$1 \le t \le 1.4$	0.4	
$1 \le t \le 1.3$	0.3	
$1 \le t \le 1.2$	0.2	
$1 \le t \le 1.1$	0.1	

(e). Do the values for the average velocity seem to be getting closer to a specific value? If so, what value?

This problem illustrates using a limit (________________________________) to find the value at one instant.

2. Suppose the revenue function for selling x desks is given by $R(x) = 25x - 0.1x^2$

(a). What is the revenue if 0 desks are sold? What is the revenue if 20 desks are sold?

(b). What is the average rate of change of revenue when going from selling 0 desks to 20 desks?

(c). What is the average rate of change of revenue when going from selling 20 desks to 40 desks?

(d). Complete the following table to determine the average rate of change when going from selling 20 desks to selling x given in the table below.

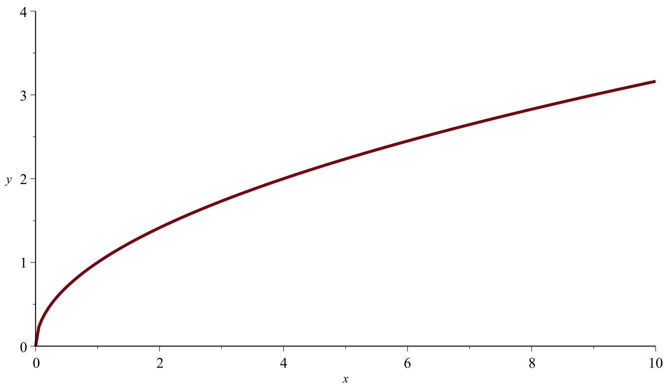
x	$R(x)$	average rate of change of revenue going from 20 to x desks.
40	$R(40) =$	$\dfrac{R(40) - R(20)}{40 - 20} =$
30	$R(30) = 660$	
25	$R(25) = 562.50$	$\dfrac{R(25) - R(20)}{25 - 20} =$
24	$R(24) = 542.40$	$\dfrac{R(24) - R(20)}{24 - 20} =$
23	$R(23) = 522.10$	$\dfrac{R(23) - R(20)}{23 - 20} = \dfrac{522.10 - 460}{3} = \dfrac{62.1}{3} = \$20.70/\text{desk}$
22	$R(22) = 501.60$	$\dfrac{R(22) - R(20)}{22 - 20} = \dfrac{501.60 - 460}{2} = \dfrac{41.6}{2} = \$20.80/\text{desk}$
21	$R(21) = 480.90$	$\dfrac{R(21) - R(20)}{21 - 20} = \dfrac{480.90 - 460}{2} = \dfrac{20.9}{2} = \$20.90/\text{desk}$

What do you think the instantaneous rate of change of the revenue is when selling 20 units?

Informal Description of a <u>Tangent Line</u> (Not the Formal Definition):

<u>The Tangent Line Problem</u>:

3. Sketch the graph of $y = \sqrt{x}$ below and label the point $P(4,2)$. Draw the tangent line to the curve at P.



(a). Let Q be the point corresponding to $x = 9$. Find the slope of the secant line PQ. $m_{PQ} =$

(b). Let Q be any point (other than P) on the curve, i.e. $Q(x, \quad)$. Find an expression (involving x) for the slope of the secant line PQ.

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

(c). Complete the table below to find the slope of the secant line PQ for the given x-coordinates of Q. Use the expression from part (b) for computing the slope. [Keep 6 decimal places.]

x	$m_{PQ} =$
1.0	
3.0	
3.5	0.258343
3.9	0.251582
3.99	0.250156
3.999	0.250016
4	??
4.001	0.249984
4.01	0.249844
4.1	0.248457
4.5	0.242641
5.0	0.236068
6.0	0.224745
9.0	

(d). Why can't we use the slope formula to compute a slope when the x-coordinate of Q is 4 (i.e. when P and Q are the same point)?

(e). Based on the table above, guess the value of the slope of the tangent line at $P(4,2)$.

(f). Use this value to write an equation of the tangent line.