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1. Differentiate the following functions.

(a). $f(x) = 6x^3 - 3x^2 + \dfrac{1}{x}$ $f'(x) = 18x^2 - 6x - \dfrac{1}{x^2}$

(b). $z = 2u^{4/3} - 3u^{3/5}$ $\dfrac{dz}{du} = \dfrac{8}{3}u^{1/3} - \dfrac{9}{5}\dfrac{1}{u^{2/5}}$

(c). $y = \sqrt{x^2 - 4x}$ $y' = \dfrac{1}{2}(x^2 - 4x)^{-1/2} \cdot (2x - 4) = \dfrac{x - 2}{\sqrt{x^2 - 4x}}$

(d). $g(t) = t\ln(4t)$ $g'(t) = 1 + \ln(4t)$

(e). $y = \sqrt{x}e^{-3x}$ $y' = x^{1/2} \cdot e^{-3x} \cdot (-3) + e^{-3x} \cdot \dfrac{1}{2}x^{-1/2} = -3\sqrt{x}e^{-3x} + \dfrac{e^{-3x}}{2\sqrt{x}}$

(f). $g(x) = \dfrac{x^2 - 3x + 1}{x + 4}$ $g'(x) = \dfrac{(x+4)(2x-3) - (x^2 - 3x + 1)(1)}{(x+4)^2} = \dfrac{x^2 + 8x - 13}{(x+4)^2}$

2. Find the equation of the tangent line to $y = 4x^2 + 2x + 5$ at $x = -1$. $y - 7 = -6(x + 1)$

3. Find the first and second derivatives of $y = (x^2 + 4)^8$ $y' = 16x(x^2 + 4)^7$ $y'' = 224x^2(x^2 + 4)^6 + 16(x^2 + 4)^7$

4. The supply of q units of a product at a price p dollars is given by $p = 25 + 145\ln(2q + 3)$

(a). Find the rate of change of supply price with respect to the number of units supplied. $\dfrac{dp}{dq} = \dfrac{290}{2q + 3}$

(b). Find the rate of change of supply price when the number of units supplied is 100. Interpret your result. 1.43
Price will go up $\approx$ \$1.43 if one more unit is supplied.

5. Given $y = \ln\left(\dfrac{x^2}{2x + 1}\right)$, use properties of logarithms to expand the function first and then find the derivative y'.
$y = 2\ln x - \ln(2x + 1) \Longrightarrow \dfrac{dy}{dx} = \dfrac{2}{x} - \dfrac{2}{2x + 1}$

6. Find the equation of the tangent line to $y = e^{x^2 - 4}$ at $x = 2$. $y - 1 = 4(x - 2)$

7. [A problem like this may be on a non-calculator portion of the exam, so do it **without** a calculator.] Given $y = x^4 - 4x^3$, use the function and its derivatives to find the **(a)** critical values, **(b)** intervals where the function is increasing and where it is decreasing, and **(c)** critical ***points***, each classified as a relative maximum, minimum, or horizontal point of inflection. Find the **(d)** intervals where the function is concave up and where it is concave down and **(e)** any inflection ***points***. Plot the points found in parts (c) and (e). Then use all of the information in parts **(a)-(e)** to sketch the function. You must show all your work for credit.
(a). $x = 0, 3$, **(b).** Increasing on $(3, \infty)$, Decreasing on $(-\infty, 0) \bigcup (0, 3)$, **(c).** Min. Pt.: $(3, -27)$, Max. Pt.: none, HPI: $(0, 0)$,
(d). Concave Up on $(\infty, 0) \bigcup (2, \infty)$ and Concave Down on $(0, 2)$, **(e).** PoI: $(0, 0)$ and $(2, -16)$, **(f).** (Use your calculator to verify)

8. Section 10.2, #27

9. A company daily sales due to an advertising campaign is given by $S = 10 + 75t + 36t^2 - t^3$
where t is the number of days after the sales campaign begins.

(a). Find how long it will take before the sales are maximized. What is the maximum sales? $t = 25$ days and max sales $= 8760$

(b). Find how long before the rate of change of sales is minimized. That is, find the point of diminishing returns. $t = 12$ days

10. Evaluate the following integrals:

(a). $\int 7x^3 - 4x + 2\ dx = \frac{7}{4}x^4 - 2x^2 + 2x + C$

(b). $\sqrt{x} - \frac{2}{x^3}\ dx = \frac{2}{3}x^{3/2} + \frac{1}{x^2} + C$

(c). $\int_1^2 x + 4x^3\ dx = \frac{1}{2}x^2 + x^4\Big|_1^2 = 16.5$

(f). $\int_0^1 (x-5)^2\ dx = \frac{1}{3}x^3 - 5x^2 + 25x\Big|_0^1 = 61/3$

11. A company finds that its marginal revenue from selling x units is given by $MR = 212 - 0.3x$. Use a definite integral to find the additional revenue gained when sales are increased from $x = 200$ units to $x = 500$ units. $32,100

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