Name: Key

Math 162, Intro to Math Methods and Applications – Crawford

Exam 2 - Form B

09 November 2016

- You may use the given formula sheet. Books or other notes (in any form) are not allowed.
- You may use a calculator, but you must show work for credit.
- ***Show all your work*** – partial credit may be given for written work.
- Clearly indicate your answers.
- Good Luck!

Calculator Number: ☐

	Score
1	/5
2	/10
3	/5
4	/10
5	/12
6	/12
7	/10
8	/12
9	/12
10	/5
11	/10
Total	/100

1. (5 pts). Find the sum of the first 98 terms of an arithmetic sequence with first term 6 and common difference $\frac{1}{2}$.

$S_{98} = \frac{98}{2}(a_1 + a_{98})$

$= \frac{98}{2}(6 + 54.5)$

$= 49(60.5)$

$= \boxed{2964.5}$

$a_{98} = a_1 + 97d$

$= 6 + 97(\frac{1}{2})$

$= 54.5$

2. (10 pts). Solve the following equations for x.

(a). $\ln(3x-4) - \ln 2 = \ln 10$

$\ln\left(\frac{3x-4}{2}\right) = \ln 10$

$e^{\ln\left(\frac{3x-4}{2}\right)} = e^{\ln 10}$

$\frac{3x-4}{2} = 10$

$3x-4 = 20$

$3x = 24$

$\boxed{x = 8}$

OR

ln(3x-4) = ln(2) + ln(10
ln(3x-4) = ln(2 x 10)
ln(3x-4) = ln(20)
3x-4 = 20
3x = 24
x = 8

(b). $9600 = 120(1.03)^x$

$\frac{9600}{120} = (1.03)^x$

$80 = (1.03)^x$

$\ln 80 = \ln(1.03)^x$

$\ln 80 = x \cdot \ln(1.03)$

$\boxed{x = \frac{\ln(80)}{\ln(1.03)} \approx 148.248}$

3. (5 pts). If \$3200 is invested for 6 months at an annual *simple* interest rate of 4%, what is the future value after 6 months?

$S = P(1+rt)$

$= 3200(1+.04(\frac{1}{2}))$

$= 3200(1.02)$

$= \boxed{\$3264.00}$

4. (10 pts). What is the future value if \$5,000 is invested for 4 years at 3%

(a). Compounded quarterly?

$S = P(1+\frac{r}{m})^{mt}$

$= 5000(1+\frac{.03}{4})^{4\cdot 4}$

$= 5000(1.0075)^{16}$

$\approx \boxed{\$5634.96}$

(b). Compounded continuously?

$S = Pe^{rt}$

$= 5000e^{.03(4)}$

$= 5000e^{.12}$

$\approx \boxed{\$5637.48}$

5. (12 pts). An individual deposits \$200 at the end of each month into an account that earns 7.2%, compounded monthly.

(a). How much will be in the account at the end of 5 years?

$R = 200 \quad r = .072 \quad m = 12 \quad i = \frac{.072}{12} = .006 \quad t = 5 \Rightarrow n = (12)(5) = 60$

$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 200\left[\frac{(1+.006)^{60}-1}{.006}\right] = \boxed{\$14{,}392.95}$

TVM Solver: N = 60, I = 7.2, PV = 0, PMT = -200, FV = ? ← Solve for FV.

P/Y = C/Y = 12

(b). If the individual wants \$25,000 in the account at the end of 5 years, how big should the monthly payments be?

$S = 25{,}000 \quad R = ?$

$S = R\left[\frac{(1+i)^n - 1}{i}\right]$

$25000 = R\left[\frac{(1+.006)^{60}-1}{.006}\right]$

$25000 = R[71.96473534]$

$R = \frac{25000}{71.96473534} \approx \boxed{\$347.39}$

TVM SOLVER:

N = 60

I = 7.2

PV = 0

PMT = ? ← Solve for PMT

FV = 25000

P/Y = C/Y = 12

6. (12 pts). Develop an amortization schedule for a loan of \$10,000 with interest at 8.5%, compounded annually, if it is to be repaid in 3 years by making 3 annual payments of equal size.

$m = 1$

Period	Payment	Interest	Balance Reduction	Unpaid Balance
	-	-	-	10000.00
1	3915.39	850.00	3065.39	6934.61
2	3915.39	589.44	3325.95	3608.66
3	3915.39	306.74	3608.65	.01
			or 3608.66	0

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$= 10000\left[\frac{.085}{1-(1+.085)^{-3}}\right] = 3915.39$$

$i = .085/1 = .085$

$n = (1)(3) = 3$

7. (10 pts). Find the following limits, if they exist. [Show work for credit.]

(a). $\lim_{x\to -1} \dfrac{9+x^2}{-2x+5} = \dfrac{9+(-1)^2}{-2(-1)+5} = \dfrac{9+1}{2+5} = \boxed{\dfrac{10}{7}}$

(b). $\lim_{x\to 4} \dfrac{x^2-x-12}{x^2-4x} = \lim_{x\to 4} \dfrac{(x-4)(x+3)}{x(x-4)} = \lim_{x\to 4} \dfrac{x+3}{x} = \dfrac{4+3}{4}$

$= \boxed{\dfrac{7}{4}}$

$\dfrac{16-4-12}{16-16}$

$\dfrac{0}{0}$

indet. form

$\Rightarrow$ More Work

8. (12 pts). Given $f(x) = 2 - 6x^2$, **use the limit definition** $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$, to show that the derivative $f'(x)$ is $-12x$. **To help with this process complete the following steps:**

(a). **Step 1.** Write down $f(x)$.

$f(x) = 2 - 6x^2$

(b). **Step 2.** Find and simplify $f(x+h)$.

$f(x+h) = 2 - 6(x+h)^2$

$= 2 - 6(x^2 + 2xh + h^2)$

$= 2 - 6x^2 - 12xh - 6h^2$

(c). **Step 3.** Find and simplify $\dfrac{f(x+h) - f(x)}{h}$. [Clearly show all algebraic steps.]

$$\frac{f(x+h) - f(x)}{h} = \frac{2 - 6x^2 - 12xh - 6h^2 - (2 - 6x^2)}{h}$$

$$= \frac{2 - 6x^2 - 12xh - 6h^2 - 2 + 6x^2}{h}$$

$$= \frac{-12xh - 6h^2}{h} = \frac{h(-12x - 6h)}{h} = -12x - 6h$$

(d). **Step 4.** Take the limit as $h \to 0$ of $\dfrac{f(x+h) - f(x)}{h}$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x\to 0} (-12x - 6h) = -12x - 6(0) = \boxed{-12x}$$

result of Step 3

For the remainder of the review sheet, use the DERIVATIVE FORMULAS, not the limit definition!

9. (12 pts). Given $f(x) = 2x^3 - 4x^2 - 5x - 4$,

(a). Find the derivative of $f(x)$.

$f'(x) = 6x^2 - 8x - 5$

(b). Find the equation of the tangent line to $f(x)$ at $x = 3$.

① pt. $y = f(3) = 2(3)^3 - 4(3)^2 - 5(3) - 4 = 2(27) - 4(9) - 15 - 4 = -1$

pt $(3, -1)$

② slope $m = f'(3) = 6(3)^2 - 8(3) - 5 = 25 = m$

Line: $y + 1 = 25(x - 3) \Rightarrow y = 25x - 75 - 1 \Rightarrow y = 25x - 76$

10. (5 pts). Find the derivative of $g(x) = \frac{5}{x^4} + 3\sqrt{x}$

$g(x) = 5x^{-4} + 3x^{1/2}$

$g'(x) = -20x^{-5} + \frac{3}{2}x^{-1/2} = \frac{-20}{x^5} + \frac{3}{2x^{1/2}}$

11. (10 pts). The profit function for producing x units is given by $P(x) = 100x - 0.2x^2 - 5000$ in dollars. Find and interpret the marginal profit for $x = 200$ units.

$MP = P'(x) = 100 - 0.4x$

$P'(200) = 100 - 0.4(200) = 20$

If one more unit is produced (i.e. the 201st unit), the profit will go up by approximately $\$20$.