Name: Key

Math 162, Intro to Math Methods and Applications – Crawford

Exam 2 - Form A

09 November 2016

Score

1	/5
2	/10
3	/5
4	/10
5	/12
6	/12
7	/10
8	/12
9	/12
10	/5
11	/10
Total	/100

- You may use the given formula sheet. Books or other notes (in any form) are not allowed.
- You may use a calculator, but you must show work for credit.
- ***Show all your work*** – partial credit may be given for written work.
- Clearly indicate your answers.
- Good Luck!

Calculator Number:

1. (5 pts). Find the sum of the first 112 terms of an arithmetic sequence with first term 4 and common difference $\frac{1}{2}$.

$$S_{112} = \frac{112}{2}(a_1 + a_{112})$$
$$= 56(4 + 59.5)$$
$$= 56(63.5)$$
$$= \boxed{3556}$$

$$a_{112} = a_1 + 111\left(\tfrac{1}{2}\right)$$
$$= 4 + 55.5$$
$$= 59.5$$

2. (10 pts). Solve the following equations for x.

(a). $6000 = 250(1.07)^x$

$\frac{6000}{250} = (1.07)^x$

$24 = (1.07)^x$

$\ln 24 = \ln(1.07)^x$

$\ln 24 = x \cdot \ln(1.07)$

$\rightarrow x = \frac{\ln 24}{\ln(1.07)}$

$\approx \boxed{46.97}$

(b). $\ln(2x-1) - \ln 3 = \ln 9$

$\ln\left(\frac{2x-1}{3}\right) = \ln 9$

$e^{\ln\left(\frac{2x-1}{3}\right)} = e^{\ln 9}$

$\frac{2x-1}{3} = 9$

$\rightarrow 2x-1 = 27$

$2x = 28$

$x = \frac{28}{2}$

$\boxed{x = 14}$

OR

ln(2x-1) = ln(3) + ln(9)
ln(2x-1) = ln(3 x 9)
ln(2x-1) = ln(27)
2x-1 = 27
2x=28
x=14

3. (5 pts). If \$1800 is invested for 6 months at an annual *simple* interest rate of 8%, what is the future value after 6 months?

$S = P(1 + rt)$

$= 1800(1 + .08(\tfrac{1}{2}))$

$= 1800(1.04)$

$= \boxed{\$1872}$

4. (10 pts). What is the future value if $10,000 is invested for 2 years at 5%

(a). Compounded quarterly?

$$S = P\left(1+\frac{r}{m}\right)^{mt}$$

$$= 10000\left(1+\frac{.05}{4}\right)^{(4)(2)}$$

$$= 10000(1.0125)^{8}$$

$$= \boxed{\$11044.86}$$

(b). Compounded continuously?

$$S = Pe^{rt}$$

$$S = 10000e^{.05(2)}$$

$$= 10000e^{.10}$$

$$= \boxed{\$11051.71}$$

5. (12 pts). An individual deposits $150 at the end of each month into an account that earns 8.4%, compounded monthly.

(a). How much will be in the account at the end of 6 years?

$R = 150$ $r = .084$ $m = 12$ $t = 6$ $\Rightarrow$ $i = \frac{.084}{12} = .007$ $n = (12)(6) = 72$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 150\left[\frac{(1+.007)^{72}-1}{.007}\right] = \boxed{\$13{,}980.55}$$

TVM SOLVER: $N = 72$ $I\% = 8.4$ $PV = 0$ $PMT = -150$ $FV = ?$ Solve for FV

$P/Y = C/Y = 12$

(b). If the individual wants $20,000 in the account at the end of 6 years, how big should the monthly payments be?

$S = 20000$ $t = 6$ $R = ?$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$20000 = R\left[\frac{(1+.007)^{72}-1}{.007}\right]$$

$$20000 = R[93.20363825]$$

$$R = \frac{20000}{93.20363825} \approx \boxed{\$214.58}$$

TVM SOLVER:

$N = 72$

$I = 8.4$

$PV = (0)$

$PMT = ?$

$FV = 20000$

$P/Y = C/Y = 12$

Solve for PMT

6. (12 pts). Develop an amortization schedule for a loan of \$30,000 with interest at 5.5%, compounded annually, if it is to be repaid in 3 years by making 3 annual payments of equal size.

$m = 1$

Period	Payment	Interest	Balance Reduction	Unpaid Balance
	-	-	-	30000.00
1	11,119.62	1650.00	9469.62	20530.38
2	11,119.62	1129.17	9990.45	10539.93
3	11,119.62	579.70	10539.92	.01

OR 10539.93 or adjust 0

Note $i = \frac{.055}{1} = .055$

$n = (1)(3) = 3$

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$= 30000\left[\frac{.055}{1-(1+.055)^{-3}}\right] \approx 11,119.62$$

7. (10 pts). Find the following limits, if they exist. [Show work for credit.]

(a). $\displaystyle\lim_{x\to -1}\frac{-3x+3}{x^2+4} = \frac{-3(-1)+3}{(-1)^2+4} = \frac{3+3}{1+4} = \boxed{\frac{6}{5}}$

(b). $\displaystyle\lim_{x\to 2}\frac{x^2-2x}{x^2+3x-10} = \lim_{x\to 2}\frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+5)} = \lim_{x\to 2}\frac{x}{x+5} = \frac{2}{2+5}$

$$= \boxed{\frac{2}{7}}$$

$$\frac{(2)^2-2(2)}{(2)^2+(3)(2)-10}$$

$$\frac{4-4}{4+6-10}$$

$$\frac{0}{0}$$

Indeterminate Form

$\Rightarrow$ More Work

8. (12 pts). Given $f(x) = 4 - 3x^2$, **use the limit definition** $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, to show that the derivative $f'(x)$ is $-6x$. **To help with this process complete the following steps:**

(a). **Step 1.** Write down $f(x)$.

$$f(x) = 4 - 3x^2$$

(b). **Step 2.** Find and simplify $f(x+h)$.

$$f(x+h) = 4 - 3(x+h)^2$$
$$= 4 - 3(x^2 + 2xh + h^2)$$
$$= 4 - 3x^2 - 6xh - 3h^2$$

(c). **Step 3.** Find and simplify $\frac{f(x+h)-f(x)}{h}$. [Clearly show all algebraic steps.]

$$\frac{f(x+h)-f(x)}{h} = \frac{4-3x^2-6xh-3h^2-(4-3x^2)}{h}$$
$$= \frac{\cancel{4}-\cancel{3x^2}-6xh-3h^2-\cancel{4}+\cancel{3x^2}}{h}$$
$$= \frac{-6xh-3h^2}{h} = \frac{\cancel{h}(-6x-3h)}{\cancel{h}} = -6x-3h$$

(d). **Step 4.** Take the limit as $h \to 0$ of $\frac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\text{result of step 3}} = \lim_{h\to 0} \underbrace{(-6x-3h)} = -6x - 3(0) = \boxed{-6x}$$

For the remainder of the review sheet, use the DERIVATIVE FORMULAS, not the limit definition!

9. (12 pts). Given $f(x) = 2x^4 - 3x^2 - 2x - 10$,

(a). Find the derivative of $f(x)$.

$f'(x) = 8x^3 - 6x - 2$

(b). Find the equation of the tangent line to $f(x)$ at $x = 2$.

① pt; $y = f(2) = 2(2)^4 - 3(2)^2 - 2(2) - 10 = 32 - 12 - 4 - 10 = 6$

ie pt $(2, 6)$

② slope: $m = f'(2) = 8(2)^3 - 6(2) - 2 = 8(8) - 12 - 2 = 50 = m$

$\boxed{y - 6 = 50(x - 2)} \Rightarrow y = 50x - 100 + 6 \Rightarrow \boxed{y = 50x - 94}$

10. (5 pts). Find the derivative of $g(x) = \dfrac{2}{x^3} + 4\sqrt{x}$

$g(x) = 2x^{-3} + 4x^{1/2}$

$g'(x) = -6x^{-4} + 4(\tfrac{1}{2})x^{-1/2}$

$\boxed{g'(x) = -6x^{-4} + 2x^{-1/2} = \dfrac{-6}{x^4} + \dfrac{2}{x^{1/2}}}$

11. (10 pts). The profit function for producing x units is given by $P(x) = 80x - 0.1x^2 - 7000$ in dollars. Find and interpret the marginal profit for $x = 500$ units.

$MP = P'(x) = 80 - 0.2x$

$P'(500) = 80 - 0.2(500) = \boxed{-20}$

If they produce one more unit (ie the 501st unit), the profit will go down by approximately \$20.