Triangles

1. Given that $x = 5 \tan \theta$, sketch right triangle involving θ and label all the sides. Then determine expressions for all 6 trigonometric functions of θ .

$$\sin \theta = \qquad \qquad \csc \theta =$$
$$\cos \theta = \qquad \qquad \sec \theta =$$
$$\tan \theta = \frac{x}{5} \qquad \qquad \cot \theta =$$

2. Given the following information, sketch and label a right triangle involving θ . In each case, write down the resulting radical expression ($\sqrt{-}$) that appears on one of the sides.

Given	Triangle	$\underline{\text{Radical Expression } (\sqrt{})}$
(a). $x = a \sin \theta$		

(b). $x = a \tan \theta$

(c).
$$x = a \sec \theta$$

(d).
$$x = \frac{a}{b}\sin\theta$$

(e).
$$x = \frac{a}{b} \tan \theta$$

(f).
$$x = \frac{a}{b} \sec \theta$$

- **3.** [Do together as class.] Consider a circle with radius a, i.e. $x^2 + y^2 = a^2$.
- (a). Sketch the top half of this circle in the xy-plane. Shade the area bounded by this semi-circle and the x-axis.

(b). What is the equation (function) for this top half of the circle.

y =

(c). Set up, but do not evaluate, the integral to find the area of this shaded region.

(d). Evaluate the integral in part (c).

Table of Trigonometric Substitutions:

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$		$1 - \sin^2 \theta = \cos^2 \theta$
$\sqrt{a^2 - b^2 x^2}$		
$\sqrt{a^2 + x^2}$		$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{a^2 + b^2 x^2}$		
$\sqrt{x^2 - a^2}$		$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{b^2 x^2 - a^2}$		