Ex Suppose $T = f(x)$ represents the temperature in a rod a position x. Suppose we take a temperature measurement and want to know the position based on that temperature.

ie. x is now a function of . Mathematically:

Def A function g is the INVERSE FUNCTION of the function f if

Notation: The inverse function is denoted by Important:

So to show that 2 functions are inverses of each other, you must show that the definition is satisfied, i.e.:

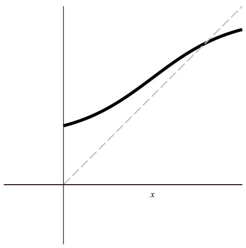
(1). Both cancelation equations are satisfied:

(2). The domain and range must interchange:
ie.

Ex Verify the $f(x) = \dfrac{1}{\sqrt{x-2}}$ has the inverse $f^{-1} = \dfrac{1}{x^2} + 2$.
(And find the domain and range of both.)

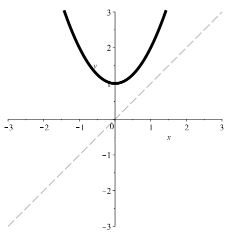
Since the domain and range interchange $\Longrightarrow$ If the point (a, b) is on f, then the point is on f^{-1}.

<u>Ex</u> Use this fact to sketch the inverse of $f(x)$.



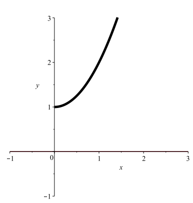
Will a function f always have an inverse function? If not, how can we tell?

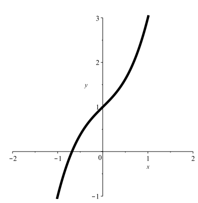
<u>Ex</u> Given the graph of $f(x) = x^2 + 1$ below, sketch its reflection through the line $y = x$.
Is the reflection a function?

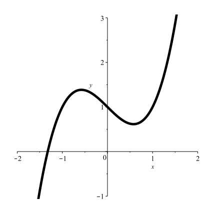


<u>Def</u> A function is called <u>ONE-TO-ONE</u> if

<u>Ex</u> Determine whether the following functions will have an inverse.







<u>Steps for finding and inverse of $f(x)$</u>

0. Verify that an inverse exists.

1. Write $y = f(x)$.

2. Solve for x in terms of y (if possible).

3. Interchange x and y and write $y = f^{-1}(x)$.

4. Define $\text{dom}(f^{-1})$ as the range of f.

<u>Ex</u> Find the inverse function of $f(x) = \sqrt{2x - 3}$