Name: Key

Math 152, Calculus II – Crawford

Exam 1

25 February 2020

- Calculators, books, notes (in any form), cell phones, and any unauthorized resources are ***not*** allowed.
- You may use the given unit circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Evaluate trigonometric, exponential, and logarithmic expressions for standard values.
- Problems #2, 4(a), and 8(a) will be used to compute extra credit for Quiz 1.
- Good Luck!

Score	
1	/8
2	/12
3	/8
4	/16
5	/10
6	/10
7	/14
8	/24
Total	/100

Formulas that you may or may not find helpful

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$\frac{d}{dx}[a^x] = a^x \cdot \ln a$ $\quad$ $\frac{d}{dx}[\log_a x] = \dfrac{1}{x \ln a}$ $\quad$ $\int a^x \, dx = \dfrac{a^x}{\ln a} + C$

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1. (8 pts). Solve the following equation for x.

$\ln(2x+1) + \ln x = 0$

$\ln(2x+1)x = 0$

$e^{\ln(2x+1)x} = e^0$

$(2x+1)x = 1$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$2x-1=0$ or $x+1=0$

$\boxed{x = \frac{1}{2}}$ $\quad$ $x = -1$

Note $x=-1$ is not allowed in original equation

2. (12 pts). Given $f(x) = \frac{1}{2}x^3 + x - 1$, find $(f^{-1})'(5)$ using the formula $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$.
[**Do not attempt to find f^{-1}.**]

$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(2)} = \boxed{\frac{1}{7}}$$

① $f^{-1}(5) = ? \Leftrightarrow f(?) = 5$

$\frac{1}{2}x^3 + x - 1 = 5$

By observation

$x = 2$

ie $f(2) = 5 \Leftrightarrow f^{-1}(5) = 2$

② $f'(x) = \frac{3}{2}x^2 + 1$

$f'(2) = \frac{3}{2}(2)^2 + 1$

$= \frac{3}{\cancel{2}}(\cancel{4}^{2}) + 1$

$= 7$

3. (8 pts). A population grows according to the model $p(t) = p_0 e^{kt}$ where p is the population at time t in years. After 3 years, the populations has grown by 24%. Find the relative growth rate k.
[You do not need a calculator. Leave you answer exact and you do not need to simplify.]

$P(3) = \cancel{p_0} e^{k \cdot 3} = 1.24 \cancel{p_0}$

$e^{3k} = 1.24$

$\ln e^{3k} = \ln(1.24)$

$3k = \ln(1.24)$

$$\boxed{k = \frac{\ln(1.24)}{3}}$$

4. (16 pts). Differentiate the following functions. [*Do not simplify.*]

(a). $y = \dfrac{xe^{-x^2}}{5-2x}$

$$y' = \frac{(5-2x)\frac{d}{dx}[xe^{-x^2}] - xe^{-x^2}\cdot\frac{d}{dx}[5-2x]}{(5-2x)^2}$$

$$= \boxed{\frac{(5-2x)[x\cdot e^{-x^2}\cdot(-2x)+e^{-x^2}\cdot 1] - xe^{-x^2}(-2)}{(5-2x)^2}}$$

(b). $h(t) = \ln\sqrt{t} - \log_4 3t$

$h(t) = \ln t^{1/2} - \log_4 3t$

$= \frac{1}{2}\ln t - \log_4 3t$

$$h'(t) = \boxed{\frac{1}{2}\cdot\frac{1}{t} - \frac{1}{3t\cdot\ln 4}\cdot 3 = \frac{1}{2t} - \frac{1}{t\cdot\ln 4}}$$

OR

$$h'(t) = \boxed{\frac{1}{t^{1/2}}\cdot\frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{3\cancel{t}\cdot\ln 4}\,3}$$

$$= \boxed{\frac{1}{2t} - \frac{1}{t\cdot\ln 4}}$$

5. (10 pts). Find the equation of the tangent line to $y = 2\cdot 3^x$ at $x = 2$. [Simplify values.]

① pt: $y = 2\cdot 3^2 = 2\cdot 9 = 18 \Rightarrow$ pt $(2, 18)$

② Slope: $y' = 2\cdot 3^x\cdot\ln 3$

$m = y'|_{x=2} = 2\cdot 3^2\ln 3 = 18\ln 3$

$$\boxed{y - 18 = 18\ln 3(x-2)}$$

6. (10 pts). Use Logarithmic Differentiation to find y' in terms of x only for

$y = x^{\cos x}$

$\ln y = \ln x^{\cos x}$

$\ln y = \cos x \cdot \ln x$

$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\cos x \cdot \ln x]$

$\frac{1}{y} y' = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x))$

$y' = \left(\frac{\cos x}{x} - (\sin x)(\ln x)\right) y$

$= \boxed{\left(\frac{\cos x}{x} - \sin x \cdot \ln x\right) \cdot x^{\cos x}}$

7. (14 pts). Evaluate the following limits.

(a). $\lim_{x\to 0} \frac{e^{4x} - 1 - 4x}{x^2}$ $\overset{L}{=} \lim_{x\to 0} \frac{4e^{4x} - 4}{2x} \overset{L}{=} \lim_{x\to 0} \frac{16e^{4x}}{2} = \frac{16}{2} = \boxed{8}$

$\frac{1-1-0}{0}$

$\frac{0}{0}$

Ind. Form

$\frac{4-4}{0} \to \frac{0}{0}$

Ind. Form

(b). $\lim_{x\to 0} (1 - x)^{3/x}$

$L = \lim_{x\to 0} (1-x)^{\frac{3}{x}}$

$\ln L = \ln \left[\lim_{x\to 0} (1-x)^{3/x}\right]$

$= \lim_{x\to 0} \ln\left[(1-x)^{3/x}\right]$

$= \lim_{x\to 0} \frac{3}{x} \ln(1-x)$

$(\to \infty \cdot 0$ Ind. Form$)$

$= \lim_{x\to 0} \frac{3\ln(1-x)}{x}$ $\frac{0}{0}$

$\overset{L}{=} \lim_{x\to 0} \frac{3 \cdot \frac{1}{1-x}(-1)}{1}$

$= \lim_{x\to 0} \frac{-3}{1-x} = -3$

ie $\ln L = -3$

$\boxed{L = e^{-3}}$

ie $\lim_{x\to 0} (1-x)^{3/x} = e^{-3}$

8. (24 pts). Evaluate the following integrals.

(a). $\int \frac{\pi}{e^{\pi x}}\,dx = \int \underbrace{\pi e^{-\pi x}\,dx}_{-du}$

$u = -\pi x$
$du = -\pi\,dx$
$-du = \pi\,dx$

$= -\int e^{u}\,du$

$= -e^{u} + C = \boxed{-e^{-\pi x} + C}$

(b). $\int \underbrace{x \sec(x^2)\,dx}_{\frac{1}{2}du}$

$u = x^2$ ✓
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \sec u\,du = \frac{1}{2}\ln|\sec u + \tan u| + C = \boxed{\frac{1}{2}\ln|\sec(x^2) + \tan(x^2)| + C}$

(c). $\int_0^{\pi} \frac{\sin x}{2 + \cos x}\,dx$ [Simplify.]

$u = 2 + \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$= -\int_{x=0}^{x=\pi} \frac{1}{u}\,du = -\ln|u|\Big|_{x=0}^{x=\pi}$

$= -\ln|2+\cos x|\Big|_0^{\pi}$

$= -\ln|2+\cos\pi| + \ln|2+\cos 0|$

$= -\ln|2-1| + \ln|3|$

$= -\cancel{\ln|1|} + \ln|3|$ 0

$= \boxed{\ln 3}$