For rational functions $f(x) = \dfrac{P(x)}{Q(x)}$,

where $P(x)$ and $Q(x)$ are polynomials:

0. If the degree of the numerator $\geq$ degree of the denominator, use long division or shortcuts to get a rational function with the higher degree polynomial in denominator.

0. $\dfrac{x^3 - 8x + 22}{x^2 + 3x - 4} = x - 3 + \dfrac{5x + 10}{x^2 + 3x - 4}$

since $x^2 + 3x - 4 \;\overline{\big)\, x^3 + 0x^2 - 8x + 22}$

Factor the denominator $Q(x)$

$$\frac{5x + 10}{x^2 + 3x - 4} = \frac{5x + 10}{(x + 4)(x - 1)}$$

1. If $Q(x)$ is a product of *distinct linear* factors

i.e. $Q(x) = (a_1x + b_1)(a_2x + b_2) \cdot \ldots \cdot (a_nx + b_3)$

then write

$$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \ldots + \frac{A_n}{a_nx + b_n}$$

1.

Ex: $\dfrac{5x + 10}{(x + 4)(x - 1)} = \dfrac{A}{x + 4} + \dfrac{B}{x - 1}$

Ex: $\dfrac{-4}{x(x + 2)(x - 2)} =$

Ex: $\dfrac{x}{2x^2 + 5x - 3} =$

2. If $Q(x)$ contains *repeated linear* factors

i.e. $Q(x)$ contains $(ax + b)^m$

then write

$$\frac{P(x)}{Q(x)} = \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \ldots + \frac{A_m}{(ax + b)^m}$$

2.

Ex: $\dfrac{3x - 1}{(x + 2)^3} = \dfrac{A}{x + 2} + \dfrac{B}{(x + 2)^2} + \dfrac{C}{(x + 2)^3}$

Ex: $\dfrac{2x^2 + 3}{(x - 1)^2(x + 1)} =$

3. If $Q(x)$ contains *distinct quadratic* factors

i.e. $Q(x)$ contains $(ax^2 + bx + c)$ which can't be factored

then for each one write a term like

$$\frac{Ax + B}{ax^2 + bx + c}$$

3.

Ex: $\dfrac{3x + 2}{x(x^2 + x + 1)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + x + 1}$

Ex: $\dfrac{-2x}{(x + 1)(x^2 + 1)} =$

4. If $Q(x)$ contains *repeated quadratic* factors

i.e. $Q(x)$ contains $(ax^2 + bx + c)^m$ which can't be factored

then write

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \ldots + \frac{A_mx + B_m}{(ax^2 + bx + c)^m}$$

4.

Ex: $\dfrac{-x^3}{(x^2 + x + 1)^2} = \dfrac{Ax + B}{x^2 + x + 1} + \dfrac{Cx + D}{(x^2 + x + 1)^2}$

Ex: $\dfrac{2x^2 + 1}{(x^2 + 1)^2} =$