

**Ex** Suppose  $T = f(x)$  represents the temperature in a rod a position  $x$ . Suppose we take a temperature measurement and want to know the position based on that temperature.

ie.  $x$  is now a function of  $T$ . Mathematically:

**Def** A function  $g$  is the INVERSE FUNCTION of the function  $f$  if

Notation: The inverse function is denoted by  $f^{-1}$ .

Important:

So to show that 2 functions are inverses of each other, you must show that the definition is satisfied, i.e.:

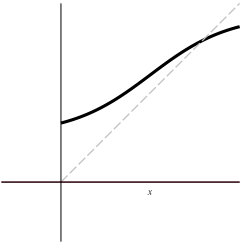
(1). Both cancelation equations are satisfied:

(2). The domain and range must interchange:  
ie.

**Ex** Verify the  $f(x) = \frac{1}{\sqrt{x-2}}$  has the inverse  $f^{-1} = \frac{1}{x^2} + 2$ .  
(And find the domain and range of both.)

Since the domain and range interchange  $\implies$  If the point  $(a, b)$  is on  $f$ , then the point  $(b, a)$  is on  $f^{-1}$ .

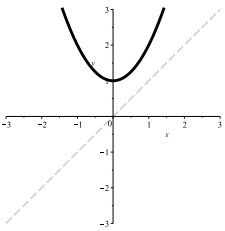
**Ex** Use this fact to sketch the inverse of  $f(x)$ .



Will a function  $f$  always have an inverse function?

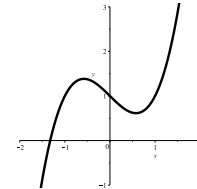
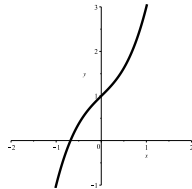
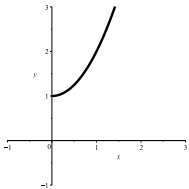
If not, how can we tell?

**Ex** Given the graph of  $f(x) = x^2 + 1$  below, sketch its reflection through the line  $y = x$ . Is the reflection a function?



**Def** A function is called ONE-TO-ONE if

**Ex** Determine whether the following functions will have an inverse.



Steps for finding and inverse of  $f(x)$

**Ex** Find the inverse function of  $f(x) = \sqrt{2x - 3}$

0. Verify that an inverse exists.
1. Write  $y = f(x)$ .
2. Solve for  $x$  in terms of  $y$  (if possible).
3. Interchange  $x$  and  $y$  and write  $y = f^{-1}(x)$ .
4. Define  $\text{dom}(f^{-1})$  as the range of  $f$ .