Lab - Sequences (Lists) and Series (Sums) Name: ______________________

Due: Friday 10/19/18 (beginning of class).

This lab will count as one homework check and you will not be allowed to drop it. You are encouraged to work with others.

1. Complete the following table for each of the sequences given:

Sequence $a_n =$	$1/n^2$	$1/n$	$1/\sqrt{n}$	$\left(\frac{1}{2}\right)^n$	$\left(-\frac{1}{2}\right)^n$	2^n	$\frac{n}{n+1}$	$\frac{n}{n^2+1}$	3
List the first 4 terms									
Find the following: $a_1 =$ $a_{10} =$ $a_{100} =$ $a_{1000} =$ $a_{10000} =$									
Do the terms seem to be approaching a particular value? If so, what number? Does the **sequence** converge or diverge?									
Series Use `sum(seq(...)` to compute the following **sums**: (keep 4 decimals) $\sum_{n=1}^{10} a_n =$ $\sum_{n=1}^{100} a_n =$ $\sum_{n=1}^{500} a_n =$ $\sum_{n=1}^{999} a_n =$									
Do the **sums** seem to be approaching a particular value? If so, what number? Does the **series** $\sum_{n=1}^{\infty} a_n$ converge or diverge?									

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2. Use the information from the table to complete the following sentences. Fill in the blanks with

"converges to #" (specify number) or *"diverges"*

The sequence $a_n = \frac{1}{n^2}$	________	and the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$	________
The sequence $a_n = \frac{1}{n}$	________	but the series $\sum_{n=1}^{\infty} \frac{1}{n}$	________
The sequence $a_n = \frac{1}{\sqrt{n}}$	________	but the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	________
The sequence $a_n = \left(\frac{1}{2}\right)^n$	________	and the series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$	________
The sequence $a_n = \left(-\frac{1}{2}\right)^n$	________	and the series $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n$	________
The sequence $a_n = 2^n$	________	and the series $\sum_{n=1}^{\infty} 2^n$	________
The sequence $a_n = \frac{n}{n+1}$	________	but the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$	________
The sequence $a_n = \frac{n}{n^2+1}$	________	but the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$	________
The sequence $a_n = 3$	________	but the series $\sum_{n=1}^{\infty} 3$	________

3. Look at all of the series (sums) that converge in the last column above. What is the limit of their corresponding sequence(list) a_n?

   Use this observation to complete the following:

   If the series $\sum_{n=1}^{\infty} a_n$ converges, then the limit of the sequence, $\lim a_n =$ ______

4. But does it work the other way? If the limit of the sequence a_n is zero (i.e. $\lim a_n = 0$), then does it guarantee that the series will converge? ***NO!!***

   State one of the examples above which shows that it does not work.

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5. Consider the **sequence** $a_n = (-1)^n$

(a) List the first 5 terms of the **sequence**.

(b) Does the limit of this **sequence** exist?

(c) Use your calculator (or your brain) to compute the following **sums**:

$\sum_{n=1}^{500}(-1)^n =$

$\sum_{n=1}^{504}(-1)^n =$

$\sum_{n=1}^{501}(-1)^n =$

$\sum_{n=1}^{505}(-1)^n =$

$\sum_{n=1}^{502}(-1)^n =$

$\sum_{n=1}^{506}(-1)^n =$

$\sum_{n=1}^{503}(-1)^n =$

$\sum_{n=1}^{507}(-1)^n =$

Do the **sums** seem to be approaching a single value?

Does the **series** $\sum_{n=1}^{\infty}(-1)^n$ converge of diverge? Why or why not?

6. Define $s_k = \sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \cdots a_k$ to be the sum of the first k terms.
Note: This is nothing new – these are the sums that you have been computing using `sum(seq(...))`.
The k just tells you when to stop adding. So for each different k you get a new number and calling it s_k.

Ex: If $a_n = \left(\frac{2}{3}\right)^n$, then $s_3 = \sum_{n=1}^{3}\left(\frac{2}{3}\right)^n = \frac{2}{3} + \frac{4}{9} + \frac{8}{27} = 1.407407407$

(a) Use the calculator to find $s_1, s_2, s_3, s_4, s_5, s_6$ for the sequence $a_n = \left(\frac{2}{3}\right)^n$. Keep 4 decimal places.

$s_1 =$ $s_2 =$ $s_3 =$

$s_4 =$ $s_5 =$ $s_6 =$

(b) Now let each s_k become terms in it's own sequence. List the values found in part (a) as the first 6 terms of the sequence below.

$\{s_k\}_{k=1}^{\infty} = \{s_1, s_2, s_3, s_4, s_5, s_6 \ldots\} =$

(c) Compute larger partial sums such as $s_{10}, s_{100}, s_{500}, s_{999}$ like you did in problem #1. Does it look like the sequence $\{s_k\}$ converges? If so, to what number?

i.e. $\lim_{k\to\infty} s_k =$ ____________. i.e. $\lim_{k\to\infty} \sum_{n=1}^{k} a_n =$ ____________. i.e. $\sum_{n=1}^{\infty} a_n =$ ____________.

7. Explain the each of the following in your own words. Consider the following questions: How are they different and how are they related? Are any of them the same thing?

   (a) a_n

   (b) $\sum_{n=1}^{k} a_n$

   (c) s_k

   (d) $\{s_k\}$

   (e) $\lim_{k\to\infty} s_k$

   (f) $\sum_{n=1}^{\infty} a_n$