1. $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12}$

Note: $\lim a_n =$ __0__ and leading order __$\frac{1}{n^2}$__

Rewrite $\dfrac{-1}{n^2+7n+12}$ in partial fraction decomposition (i.e. go backwards from common denominator)

(a). Factor denominator $\dfrac{-1}{n^2+7n+12} = \dfrac{-1}{(n+3)(n+4)}$

(b). "Break up" fraction: $\dfrac{-1}{(n+3)(n+4)} = \dfrac{A}{n+3} + \dfrac{B}{n+4}$ Need to determine A and B .

(c). Recombine with __common denominator__ : $\dfrac{-1}{(n+3)(n+4)} = \dfrac{A(n+4)+B(n+3)}{(n+3)(n+4)}$

Numerators must be equal: $-1 = An + 4A + Bn + 3B = (A+B)n + (4A+3B)$

Collect Terms:	LHS	= RHS
n:	0	$= A + B$
constant:	-1	$= 4A + 3B$

Solve for A and B

(d). Then $\dfrac{-1}{(n+3)(n+4)} = \dfrac{-1}{n+3} + \dfrac{1}{n+4}$

So does $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12} = \sum_{n=1}^{\infty}\left(\frac{-1}{n+3} + \frac{1}{n+4}\right)$ converge? If so, to what?

Look at partial sums s_k:

$$
\begin{aligned}
s_1 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)\\
s_2 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)\\
s_3 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right)\\
s_4 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right)+\left(-\tfrac{1}{7}+\tfrac{1}{8}\right)\\
&\vdots\\
s_k &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right)+\left(-\tfrac{1}{7}+\tfrac{1}{8}\right)+\cdots+\left(-\tfrac{1}{k+2}+\tfrac{1}{k+3}\right)+\left(-\tfrac{1}{k+3}+\tfrac{1}{k+4}\right)\\
s_k &= -\tfrac{1}{4}+\tfrac{1}{k+4}
\end{aligned}
$$

So $\lim s_k = -\frac{1}{4} + 0 = -\frac{1}{4}$. Thus the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12} = -\frac{1}{4}$

since __the sequence of partial sums $s_k \Rightarrow -\frac{1}{4}$__ .

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

$$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} \Longrightarrow$$

$$\Longrightarrow 3 = (A+B)n + 3A$$

Collect Terms:	LHS	= RHS	
n:	0	$= A + B$	$A = 1 \quad B = -1$
constant:	3	$= 3A$	

So $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3}$

$$\begin{aligned}
s_1 &= \left(1 - \tfrac{1}{4}\right) \\
s_2 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) \\
s_3 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) \\
s_4 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) \\
s_5 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) \\
s_6 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right) \\
s_7 &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right) + \left(\tfrac{1}{7} - \tfrac{1}{10}\right) \\
&\vdots \\
s_k &= \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right) + \left(\tfrac{1}{7} - \tfrac{1}{10}\right) + \cdots \\
&\quad + \cdots + \left(\tfrac{1}{k-3} - \tfrac{1}{k}\right) + \left(\tfrac{1}{k-2} - \tfrac{1}{k+1}\right) + \left(\tfrac{1}{k-1} - \tfrac{1}{k+2}\right) + \left(\tfrac{1}{k} - \tfrac{1}{k+3}\right) \\
s_k &= 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{k+1} - \frac{1}{k+2} - \frac{1}{k+3}
\end{aligned}$$

Then $\lim s_k = 1 + \dfrac{1}{2} + \dfrac{1}{3} - 0 - 0 - 0 = \dfrac{11}{6}$

Therefore the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ converges to $\dfrac{11}{6}$