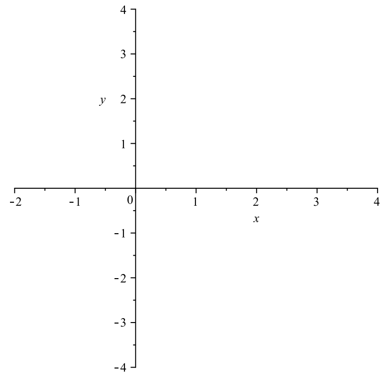
**<u>Ex</u>** Given $x = \dfrac{3}{t^2+1}$ and $y = t - 1$




Graph the curve on your calculator for $-5 \le t \le 5$.
Use a viewing window $-2 \le x \le 4$ and $-4 \le y \le 4$.

Sketch a copy on the axes to the right.

Then answer the questions below.

**(a).** Does this curve represent a function? Why or why not?

**(b).** Could you draw tangent lines to this curve? If so, sketch the tangent line at the point $\left(\frac{3}{2}, 0\right)$.

Give an estimate of the slope of the tangent line that you just drew.

**(c).** Based on part (b), does it make sense to talk about the derivative $\frac{dy}{dx}$ (or slope) of this curve?
Note: This is the derivative of $y$ with respect to $x$.

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So given $x = x(t)$ and $y = y(t)$ how do we find $\dfrac{dy}{dx}$?

<u>Solution</u>: Suppose we *could* eliminate $t$ and write ____________ $(*)$, then $\dfrac{dy}{dx} =$ ____________ $(**)$

But $y = y(t)$ and $x = x(t)$, so $(*)$ becomes $y(t) =$

Differentiate w/ respect to t: $\frac{d}{dt}[y(t)] = \frac{d}{dt}[F(x(t))]$

Recall, from $(**)$, $\dfrac{dy}{dx} = F'(x) \quad \Rightarrow$ In Leibniz Notation:

[Note: Formula is still valid even if we can't eliminate $t$ and write $y = F(x)$.]

**Ex** Given $x = \dfrac{3}{t^2+1}$ and $y = t - 1$

**(a)**. Find $\dfrac{dy}{dx}$.

**(b)**. Find the slope of the tangent line to the curve at the point $\left(\frac{3}{2}, 0\right)$.

**(c)**. Find the equation of the tangent line to the curve at the point $\left(\frac{3}{2}, 0\right)$.

**(d)**. Graph this tangent line on your calculator along with the original curve to verify that it is, in fact, the correct tangent line. [Hint: Remember you need to write the tangent line as a parametric curve.]

**(e)**. Find the equation of the tangent line to the curve at the point corresponding to $t = 0$.

Can we find $2^{nd}$ derivatives? i.e. How do we find $\dfrac{d^2y}{dx^2}$?

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$= \frac{d}{dx}\left[y'\right]$$

$$= \frac{d(y')}{dx}$$

but $y'$ is parametrically defined (e.g. function of $t$), so use the formula from p.1 replacing $y$ with $y'$.

$=$

$=$

$=$

Warning!! $\dfrac{d^2y}{dx^2} \neq$

**Ex** $x = \cos 5t, \quad y = \sin 5t$

**(a)**. Find $\dfrac{dy}{dx}$.

**(b)**. Find $\dfrac{d^2y}{dx^2}$.

**Ex** $x = t^3 - 6t^2, \qquad y = t^3 - 12t$

**(a)**. Find the points on the curve where the tangent is horizontal or vertical.

**(b)**. For what values of $t$ is the ***curve*** increasing or decreasing?

**(c)**. Use the information obtained in parts (a) and (b) to help sketch the curve on the axes below.