Name: Key

Math 152 Calculus II – Crawford

Quiz 4

04 December 2018

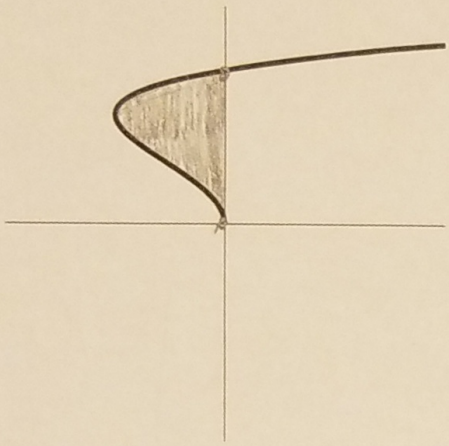
Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (5 pts) Given the parametric curve $x = t^2 - 2t$ and $y = \sqrt{t}$ [graphed below],

(a). Shade the region bounded by the parametric curve and the y-axis.

(b). Set up, ***but do not evaluate***, the integral(s) to find the area enclosed by the parametric curve and the y-axis.

[Be sure to include correct bounds.]



Area between curve & y-axis

$$A = \int_{y_1}^{y_2} (x_2 - x_1)\, dy$$

0 - x(t)

$$= \int_a^b (-x(t))\, y'(t)\, dt$$

t-values

Bounds when $x = 0$

$x = t^2 - 2t = 0$

$t(t-2) = 0$

$t = 0, 2$

$$= \boxed{\int_0^2 -(t^2-2t)\cdot \tfrac{1}{2} t^{-1/2}\, dt}$$

2. (10 pts) Given $x = 2 - e^{3t}$ and $y = t^2 + t$,

(a). Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{-e^{3t}\cdot 3} = \frac{2t+1}{-3e^{3t}}$$

(b). Find $\frac{d^2y}{dx^2}$. [Do not simplify.]

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{2t+1}{-3e^{3t}}\right]}{dx/dt}$$

$$= \frac{\frac{-3e^{3t}(2) - (2t+1)(-9e^{3t})}{(-3e^{3t})^2}}{-3e^{3t}}$$

$$= \frac{-3e^{3t}(2 - 3(2t+1))}{(-3e^{3t})^2}$$

$$= \frac{-6t-1}{9e^{6t}}$$ simplified.

(c). Find an equation for the tangent line at the point $(1, 0)$:

① pt $(1,0)$

which occurs when

$x = 2 - e^{3t} = 1$ } $t = 0$ by observation

$y = t^2 + t = 0$ }

② slope $m = \left.\frac{dy}{dx}\right|_{t=0}$

$$= \frac{2t+1}{-3e^{3t}} = \frac{0+1}{-3e^0} = \frac{1}{-3}$$

$$y - 0 = -\frac{1}{3}(x-1)$$