Name: Key

Math 152 Calculus II – Crawford

Quiz 3

30 October 2018

Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (4 pts) Determine whether the following series is convergent or divergent. If it is convergent, find the sum.

$$\sum_{n=1}^{\infty} 2^{n-1} 5^{-n} = \sum_{n=1}^{\infty} \frac{2^n 2^{-1}}{5^n} = \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{2}{5}\right)^n$$

$|r| = \frac{2}{5} < 1$

$\therefore$ Converges by the Geometric Series Test

Converges $\Rightarrow$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{2}{5}\right)^n = \frac{\frac{1}{2}\left(\frac{2}{5}\right)}{1-\frac{2}{5}} = \frac{\left(\frac{1}{5}\right)}{\left(\frac{3}{5}\right)} = \frac{1}{\cancel{5}} \cdot \frac{\cancel{5}}{3}$

$$= \boxed{\frac{1}{3}}$$

$\displaystyle\sum_{n=m}^{\infty} ar^n = \frac{ar^m}{1-r}$ if $|r| < 1$

2. (11 pts) Determine whether the following series converge or diverge. [Show all your work and clearly indicate any tests that you use.]

(a). $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{2n^2+4n}{9n^2+1}}$

$$\lim_{n\to\infty} \sqrt{\frac{2n^2+4n}{9n^2+1}} = \sqrt{\frac{2}{9}} \neq 0$$

$\boxed{\text{Series diverges by test for Divergence}}$

[Continued on the next page ⟶]

[Continued from previous page.]

Determine whether the following series converge or diverge. [Show all your work and clearly indicate any tests that you use.]

(b). $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

Consider $\int_1^\infty x^2 e^{-x^3}\,dx$

$u = -x^3$ ✓

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$= \int_{x=1}^{x=\infty} -\frac{1}{3} e^u \,du$

$= -\frac{1}{3} e^u \Big|_{x=1}^{x=\infty} = -\frac{1}{3} e^{-x^3} \Big|_1^\infty$

$= \lim_{x\to\infty} -\frac{1}{3} \underbrace{e^{-x^3}}_{0} + \frac{1}{3} e^{-(1)^3} = \frac{1}{3} e^{-1}$ ← integral converges

Since the integral converges, $\sum n^2 e^{-n^3}$ converges by the Integral Test.

(c). $\sum_{n=1}^{\infty} \frac{2n^2 - 3n}{(1+n^2)^2}$

L.D. $\frac{n^2}{(n^2)^2} = \frac{n^2}{n^4} = \frac{1}{n^2}$

$2n^2 - 3n < 2n^2$

and $\frac{1}{(1+n^2)^2} < \frac{1}{(n^2)^2}$

Combine $\Rightarrow \frac{2n^2-3n}{(1+n^2)^2} < \frac{2n^2}{n^4} = \frac{2}{n^2}$

And since $\sum \frac{2}{n^2} = 2\sum \frac{1}{n^2}$ is a converging p-series $(p = 2 > 1)$, then $\sum \frac{2n^2-3n}{(1+n^2)^2}$ converges by the Comp. Test.

OR $b_n = \frac{1}{n^2}$ $\lim_{n\to\infty} \frac{\left(\frac{2n^2-3n}{(1+n^2)^2}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{2n^2-3n}{1+2n^2+n^4} \cdot \frac{n^2}{1}$

$= \lim_{n\to\infty} \frac{2n^4 - 3n^3}{\ldots} = 2$ which is finite

Then since $\sum \frac{1}{n^2}$ converges (p-series)

[illegible]