New material: Section 11.10, 10.1-10.4

1. Find a Taylor series for $f(x) = \sqrt{x}$ centered at $a = 4$.

$f(x) = 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-3)}{2^{3n-1} n!}(x-4)^n$

2. Use a known Maclaurin series to find the Maclaurin Series for $f(x) = e^{x/2}$

$e^{x/2} = \sum_{n=0}^{\infty} = \frac{(x/2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$

3. Use a known Maclaurin series to evaluate $\int \sin(x^2)\, dx$ as an infinite series.

$C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{4n+3}}{4n+3}$

4. Find the sum of the series $\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$.

$\sum_{n=0}^{\infty} \frac{(x^4)^n}{n!} = e^{x^4}$

5. Given the parametric equations: $x = \ln t, \quad y = 1 + t^2$,

(a). Find dy/dx. $dy/dx = 2t^2$

(b). Find d^2y/dx^2. $d^2y/dx^2 = 4t^2$

(c). Find the equation of the tangent line at the point (0,2). $y = 2x + 2$

(d). Eliminate the parameter t to find the Cartesian equation of the curve. Express your answer in the form $y = f(x)$. $y = 1 + e^{2x}$

6. Given the parametric curve: $x = \sin 2t \;\; y = 4 \sin t$ on $0 \le t \le 2\pi$, find all the points where there is a horizontal or vertical tangent line. [You must show all work!]

Hor: $t = \pi/2, 3\pi/2 \Rightarrow$ pts. $(0, 4), (0, -4)$;

Ver: $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \Rightarrow$ pts. $(1, 2\sqrt{2}), (-1, 2\sqrt{2}), (1, -2\sqrt{2}), (-1, -2\sqrt{2})$

7. Find the area of the region bounded by the parametric curve $x = 2 \cot\theta, y = 2\sin^2\theta$ for $0 < \theta < \pi$.
[Be careful, the curve is traced out from right to left.] 4π

8. Sec. 10.1 #28

I(b), II(c), III(f), IV(e), V(a), VI(d)

9. For each of the polar coordinates $\left(-1, \frac{\pi}{3}\right)$ and $(2, 3\pi)$,

(a). Plot them in the polar coordinate system.

(b). Find the Cartesian coordinate. $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ and $(-2, 0)$

10. Sec. 10.3 #54

I(f), II(c), III(b), IV(d), V(e), VI(a)

11. Find a polar equation for the curve given by the Cartesian equation $x + y = 2$

$r = \frac{2}{\cos\theta + \sin\theta}$

12. Identify the curve by finding a Cartesian equation for the curve $r = 4 \sec\theta$

It is the vertical line $x = 4$.

13. Find the slope of the tangent line to the polar curve $r = \sin 3\theta$ at $\theta = \frac{\pi}{6}$.

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = -\sqrt{3}$

14. Find the points on the curve $r = 2\cos\theta$ where the tangent line is horizontal or vertical for $0 \le \theta < \pi$.

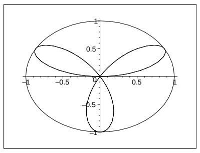
Horiz: $(\sqrt{2}, \frac{\pi}{4}), (-\sqrt{2}, \frac{3\pi}{4})$ Vert: $(2, 0), (0, \frac{\pi}{2})$

15. Find the **points** of intersection of the following curves. $r = \sin\theta, \quad r = \sin 2\theta$. (Why is it sufficient to only consider the interval $[0, 2\pi]$?)

polar coordinates $(0, 0), (0, \pi), (0, 2\pi), (\sqrt{3}/2, \pi/3), (-\sqrt{3}/2, 5\pi/3)$

16. Set up, but do not evaluate, the integral(s) to find the area inside the curve $r = 1$ and outside the curve $r = \sin 3\theta$. [Hint: Use symmetry.]

$A = A_{circle} - A_{rose} = \pi(1)^2 - \int_{-\pi/3}^{\pi/3} \frac{1}{/} 2(\sin 3\theta)^2$



17. Section 10.4 #29

<u>THE REMAINDER OF THIS REVIEW COVERS MATERIAL UP TO AND INCLUDING SECTION 11.10.</u>

18. Use the geometric series to expand $f(x) = \dfrac{1}{1+2x}$ as a power series.

$\sum_{n=0}^{\infty} (-2x)^n = 1 - 2x + 4x^2 - 8x^3 + \cdots$

19. For the function $f(x) = 1 + x + x^2$,

(a). Find the Taylor Series for $f(x)$ centered at $a = 2$.

$f(x) = 7 + 5(x-2) + \frac{2(x-2)^2}{2!}$ OR $f(x) = 1 + x + x^2$

(b). Expand and simplify your answer to **(a)**. Explain why this simplified expression makes sense.

The Taylor series for any polynomial <u>**is**</u> the polynomial, since all higher order derivatives are 0.

[Section 11.10 for more practice finding Taylor Series.]

20. Section 11.10 #55

21. Find the radius of convergence and the exact <u>interval</u> of convergence for the following series.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!\,(x-3)^n}{2^n}$ $R = 0$ and converges only at $x = 3$

(b). $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n2^n}$ $R = 2$ and $(-4, 0]$.

22. Explain the Integral Test in your own words. Include sketches to illustrate how the series and the integral are related.

23. Find the <u>**SUM**</u> of the following series or show that it diverges.

(a). $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n+1}}{5^n} = 10$

(b). $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right) = -1/2$

24. Determine whether the following series diverge or converge.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{1+n^2}$ The absolute values converge by Comparison Test . Therefore series converges (absolutely).

(b). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ Diverges (Limit Comparision Test).

(c). $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^3+1}$ Converges (Alternating Series Test)

[See old Review Sheets, Exam 3, and Chapter 11 for more practice with series tests.]

25. Error bounds for alternating series and for series that converge by the Integral Test. [See Sections 11.3 & 11.5.]

26. Determine whether the following <u>sequences</u> converge or diverge. Find the limit if it converges.

(a). $a_n = \dfrac{10^n}{9^{n+1}}$ Diverges (to ∞).

(b). $a_n = \dfrac{2n^3 - 1}{3 + n^3}$ Converges to 2

27. Find the derivative.

(a). $f(\theta) = e^{\sin 2\theta} + 3^\theta$ $f'(\theta) = 2\cos 2\theta e^{\sin 2\theta} + (\ln 3)3^\theta$

(b). $y = \sin^{-1}(x^2)$ $y' = \frac{2x}{\sqrt{1-(x^2)^2}}$

(c). $y = \sinh x$ $dy/dx = \cosh x$

(d). $y = (\sin x)^{3x}$ $y' = (3x\cot x + 3\ln(\sin x))\,(\sin x)^{3x}$

28. Find the equation of the tangent line to $y = \log_3 x$ at $x = 9$. $y - 2 = \frac{1}{9\ln 3}(x-9)$

29. Evaluate the following limits:

(a). $\lim_{x\to\infty}(1+2x)^{1/3x}$ $=1$

(b). $\lim_{x\to 0}\frac{\sin x}{x^2}$ DNE, since one-sided limits are different

30. Evaluate the following integrals:

(a). $\int \sec^4(3x)\tan^2(3x)\,dx$ $= \frac{1}{15}\tan^5(3x) + \frac{1}{9}\tan^3(3x) + C$

(b). $\int \frac{1}{x(\ln x)^2}\,dx = -\frac{1}{\ln x} + C$

(c). $\int \frac{4x}{(x^2-1)(x^2+1)}\,dx$
$= \ln|x-1| - \ln|x+1| - \ln|x^2+1| + C$

(d). $\int \frac{x}{x^2-x}\,dx = \ln|x-1| + C$

(e). $\int_0^2 \frac{1}{x^2-1}\,dx$ Improper: $\int_0^1 \frac{1}{x^2-1}\,dx + \int_1^2 \frac{1}{x^2-1}\,dx$: Div

(f). $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx = \frac{40}{3}$

(g). $\int \frac{1}{x^3-4x^2+4x}\,dx$
$= \frac{1}{4}\ln|x| - \frac{1}{4}\ln|x-2| - \frac{1}{2}\frac{1}{x-2} + C$

(h). $\int 4^t\,dt = \frac{4^t}{\ln 4} + C$

31. Find the area under the curve $f(x) = \frac{1}{x^2+16}$ for $0 \le x \le 3$. $\frac{1}{4}\tan^{-1}\left(\frac{3}{4}\right)$

32. Integrate $\int_0^\infty xe^{ax}\,dx$ for $a \neq 0$ and determine for which values of a it converges. $\left.\frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax}\right|_0^\infty$ converges when $a < 0$.

33. Section 6.5 #3

34. Use Simpson's Rule with $n = 4$ to approximate the value of $\int_4^6 \frac{1}{x^2}\,dx$. $\int_4^6 \frac{1}{x^2}\,dx \approx \frac{1}{6}\left[\frac{1}{16} + 4\cdot\frac{4}{81} + 2\cdot\frac{1}{25} + 4\cdot\frac{4}{121} + \frac{1}{36}\right]$
[Do **NOT** simplify.]

35. Use the Trapezoid Rule with $n = 6$ to approximate the value of $\int_0^{10}\sin(x^2)\,dx$. [Do **NOT** simplify.]
$T_6 = \frac{1}{2}\cdot\frac{5}{3}\left[\sin(0) + 2\sin(5/3) + 2\sin(10/3) + 2\sin(5) + 2\sin(20/3) + 2\sin(25/3) + \sin(10)\right]$

36. Let g denote the inverse function of f i.e. $g = f^{-1}$. Given $f(x) = 3x + \cos 2x$ on $0 \le x \le \frac{\pi}{2}$, find $g'(1)$. $\frac{1}{3}$

37. Find the exact value of the following:

(a). $\sin\left(\arctan\frac{5}{4}\right) = \frac{5}{\sqrt{41}}$

(b). $\arctan\left(\sin\frac{3\pi}{2}\right) = -\frac{\pi}{4}$

(c). $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = -\frac{\pi}{4}$

38. Solve the following equations for x. [Simplify your answers.]

(a). $\ln 2 + \ln(x-3) = 4$ $x = \frac{1}{2}e^4 + 3$

(b). $e^{x^2+x} = 1$ $x = 0, -1$

[Look at previous exams, quizzes, and review sheets for more review.]