1. Find the **SUM** of the following series or show that it diverges.

(a). $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}+(-1)^n\right)$ Diverges by Test for Divergence

(b). $\displaystyle\sum_{n=0}^{\infty}\frac{3^{n-1}}{5^{n+2}}=\frac{1}{30}$

(c). $\displaystyle\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)=\frac{3}{2}$

(d). $\displaystyle\sum_{n=1}^{\infty}\frac{3^{2n}}{4^n}$ Diverging Geometric Series

2. Given $\displaystyle\sum_{n=1}^{\infty}\frac{4}{n^3}$

(a). Find the 3rd partial sum s_3. $\displaystyle s_5=\sum_{n=1}^{3}4\frac{1}{n^3}=\frac{251}{54}\approx 4.648$

(b). If s_3 is used to approximate the infinite series, what is the bound for the maximum possible error? (i.e. bound on $|R_3|$?) $\displaystyle R_3\leq\int_3^{\infty}\frac{4}{x^3}\,dx=\frac{2}{9}$

(c). How many terms are needed for error to be less than 0.001. 45 terms

3. Determine whether the following **series** converge or diverge. Show all your work and clearly indicate any tests that you use.

(a). $\displaystyle\sum_{n=1}^{\infty}\frac{5n^2+n}{3-2n^2}$ Diverges by the Test for Divergence

(d). $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2 3^n}{(2n)!}$ Converges by Ratio Test

(b). $\displaystyle\sum_{n=1}^{\infty}\frac{5\sqrt{n}+1}{3+2n^2}$ Converges by Limit Comparison Test

(e). $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2n-1}\right)^{3n}$ Converges by the Root Test

(c). $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n}$ Diverges by Integral Test

(f). $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$ Since $\displaystyle\sum_{n=1}^{\infty}\left|\frac{\cos n}{n^2}\right|$ converges by the Comparison Test (needed positive values), then $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$ converges (absolutely) by Absolute Convergence Test

[More practice can be found in Section 11.7 and the Chapter 11 Review.]

4. Does $\displaystyle\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1}{\ln n}$ converge absolutely, converge conditionally, or diverge. Converges Conditionally since $\displaystyle\sum_{n=2}^{\infty}\left|(-1)^{n-1}\frac{1}{\ln n}\right|=\sum_{n=2}^{\infty}\frac{1}{\ln n}$ diverges (by the Comparison Test), but $\displaystyle\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1}{\ln n}$ converges (by the Alternating Series Test).

5. Given $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n+4}$

(a). Find the 5th partial sum s_5. $s_5 = \sum_{n=0}^{5}(-1)^n \frac{1}{n+4} \approx 0.0877$

(b). If s_5 is used to approximate the infinite series, what is the bound for the maximum possible error? (i.e. bound on $|R_5|$?) $|R_5| < b_6 = \frac{1}{6+4} = \frac{1}{10} = 0.1$

(c). How many terms are needed for error to be less than 0.001. 996 terms

6. Find the interval and radius of convergence for the following series.

(a). $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^n}$ $R = \infty$ $(-\infty, \infty)$

(b). $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$ $R = 1$ $(-3, -1]$

(c). $\sum_{n=1}^{\infty} \frac{(2x+4)^n}{n4^n}$ $R = 2$ $[-4, 0)$

7. Find the radius of convergence for the following series.

(a). $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n}$ $R = \frac{1}{3}$

(b). $\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$ $R = \frac{1}{e}$

8. Use a known power series to find a power series representation for the $f(x) = \frac{1}{1+3x^2}$. $= \sum_{n=0}^{\infty}(-1)^n 3^n x^{2n}$