1. Find the following limits. Clearly show all steps and indicate where you use L'Hopital's Rule.

(a). $\lim_{x\to 0} \frac{\sin 4x}{2x} = 2$ **(b).** $\lim_{x\to\infty} (1+x)^{\frac{1}{x}} = 1$ **(c).** $\lim_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$ **(d).** $\lim_{x\to 2} \frac{x-2}{x^2-3x-2} = 0$

2. Evaluate the following integrals

(a). $\int_0^{\pi/2} \sin(9x)\cos(x)\,dx = \frac{1}{10}$

(b). $\int \frac{-1}{\sqrt{x^2-2x-3}}\,dx = -\ln\left|\frac{x-1}{2} + \frac{\sqrt{(x-1)^2-4}}{2}\right| + C$

(c). $\int \tan^3 x \sec^4 x\,dx = \frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

(d). $\int \frac{1}{x^3-4x^2+4x}\,dx = \frac{1}{4}\ln|x| - \frac{1}{4}\ln|x-2| - \frac{1}{2}\frac{1}{x-2} + C$

(e). $\int x^3(\ln 2x)\,dx = \frac{1}{4}x^4\ln(2x) - \frac{1}{16}x^4 + C$

(f). $\int \sqrt{9-x^2}\,dx = \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x\sqrt{9-x^2}}{2} + C$

(g). $\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + C$

(h). $\int \frac{4x}{(x^2-1)(x^2+1)}\,dx = \ln|x-1| + \ln|x+1| - \ln|x^2+1| + C$

3. Integrate: $\int e^x \sin 2x \, dx = \frac{1}{5}\left(e^x \sin 2x - 2e^x \cos 2x\right) + C$

4. Approximate the integral $\int_4^6 \frac{1}{x^2} \, dx$, using [Do **NOT** simplify.]

(a). Simpson's Rule with $n = 4$

$$\int_4^6 \frac{1}{x^2} \, dx \approx \frac{1}{6}\left[\frac{1}{16} + 4 \cdot \frac{4}{81} + 2 \cdot \frac{1}{25} + 4 \cdot \frac{4}{121} + \frac{1}{36}\right]$$

(b). The Trapezoid Rule with $n = 6$

$$\int_4^6 \frac{1}{x^2} \, dx \approx \frac{1}{6}\left[\frac{1}{16} + 2 \cdot \frac{9}{169} + 2 \cdot \frac{9}{196} + 2 \cdot \frac{1}{25} + 2 \cdot \frac{9}{256} + 2 \cdot \frac{9}{289} + \frac{1}{36}\right]$$

5. Evaluate the following improper integral: $\int_0^\infty (7x - 3)e^x \, dx$ Diverges to $+\infty$.

6. Evaluate the following integrals or show that it is a divergent improper integral.

(a). $\int_0^1 \frac{1}{4 - 2x} \, dx = \frac{1}{2} \ln 2$ **(b).** $\int_0^4 \frac{1}{4 - 2x} \, dx$ Diverges **(c).** $\int_{-\infty}^\infty 2x^2 e^{-x^3} \, dx$ Diverges

7. Find the area bounded by $f(x) = \sin^2 x \cos^2 x$ on the interval $[0, \pi/2]$.

$$A = \frac{1}{8}x - \frac{1}{32}\sin 4x\bigg|_0^{\pi/2} = \frac{\pi}{16}$$

8. Write out the ***form*** of the partial fraction decomposition for the following function. Do **NOT** determine the values of the coefficients.

$$\frac{3x - 4}{x^2(x-2)(x^2+9)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2} + \frac{Hx+I}{(x^2+9)^3}$$

9. Use the Comparison Theorem for integrals to determine whether the following integral converges or diverges.

$$\int_1^\infty \frac{\arctan x}{x^{3/2}} \, dx$$

Since $\frac{\arctan x}{x^{3/2}} < \frac{\frac{\pi}{2}}{x^{3/2}}$ and $\int_1^\infty \frac{\frac{\pi}{2}}{x^{3/2}} \, dx = \frac{\pi}{2}\int_1^\infty \frac{1}{x^{3/2}} \, dx$ converges ($p = \frac{3}{2} > 1$), then $\int_1^\infty \frac{\arctan x}{x^{3/2}} \, dx$ converges by comparison.

10. Determine whether the following <u>**sequences**</u> converge or diverge. Find the limit if it converges.

(a). $a_n = \frac{1}{n} + (-1)^n$ Div./DNE **(b).** $a_n = \frac{1}{5^n} \to 0$ **(c).** $b_n = \frac{\cos n}{n} \to 0$ **(d).** $\left\{\frac{5n^2 + n}{3 - 2n^2}\right\}_{n=1}^\infty \to -\frac{5}{2}$