1. Given $f(x) = \sin x + 2\cos x$, $-\frac{\pi}{4} \le x \le 0$, find $\left(f^{-1}\right)'(a)$ for $a = 2$.

2. Solve for x: **(a).** $e^{3x-7} = 6$ **(b).** $\ln(x+1) - \ln x = 1$

3. Given $f(x) = \ln(\cos x + 2)$

(a). What is the domain of f?

(b). Find the relative extreme values on the interval [-1,1]. [Indicate max or min.]

4. Evaluate the following integrals.

(a). $\displaystyle\int_0^1 \frac{1}{4-2x}\,dx$

(b). $\displaystyle\int e^{-7x} - \frac{7\ln x}{x}\,dx$

(c). $\displaystyle\int \frac{3x^2+2x}{x^3+x^2+1}\,dx$

(d). $\displaystyle\int \frac{3x}{\sqrt{1-36x^2}}\,dx$

(e). $\displaystyle\int \frac{3}{\sqrt{1-36x^2}}\,dx$

(f). $\displaystyle\int \operatorname{sech}^2(5x)\,dx$

(g). $\displaystyle\int \sec(5x)\,dx$

(h). $\displaystyle\int_0^{\ln 2} \frac{e^{3x}+1}{e^x}\,dx$
Simplify using log./exp. properties.

(i). $\displaystyle\int \frac{\tan x}{\ln(\cos x)}\,dx$

5. Given the function $f(x) = e^{-x^2}$

(a). Evaluate $\displaystyle\lim_{x\to-\infty} e^{-x^2}$ **(b).** Evaluate $\displaystyle\lim_{x\to+\infty} e^{-x^2}$ **(c).** Find $f'(x)$

6. What is the formula for $\log_a x$ in terms of the natural logarithmic function?

7. Find the exact value of **(a).** $\sin(2\tan^{-1}(1))$ **(b).** $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

8. Simplify the following expression so that it is an algebraic expression of x: $\cos(\sin^{-1}(2x))$

9. Differentiate the following functions:

(a). $y = x^4 - 4^x + e^{4x} + \ln 4x$

(b). $y = x^{\frac{1}{x}}$

(c). $y = \pi^x - \ln e^x$

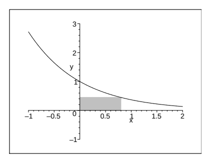
(d). $h(\theta) = 3\ln\left(\frac{2+\cos\theta}{\theta^2}\right)$

(e). $y = \cosh(\ln(x^3 + 2x^2))$

(f). $f(x) = x\sin^{-1}(3x)$

10. Find the equation of the tangent line to $y = \log_3 x$ at $x = 1$.

11. (a). Find the maximum area of a rectangle in the first quadrant with 2 sides on the x- and y-axes and one vertex on the curve $y = e^{-x}$. See the figure below. [Hint: Express the area of such a rectangle in terms of x only.]



(b). Sketch the picture for a rectangle in the first quadrant with 2 sides on the x- and y-axes and one vertex on the curve $y = e^x$. Without using Calculus, determine whether there exists such a rectangle with a maximum area. Briefly explain (a couple of sentences) why or why not.

12. Given that a population follows the law of exponential growth, $y(t) = Ce^{kt}$ where y is the population and t is time in years.

(a). Find the proportional constant k, if the population triples every 50 years.

(b). If the population is 100 after 5 years, find the population at time t.