Name: Key

Math 152, Calculus II – Crawford

Exam 1

18 September 2018

- No calculators, books, or notes (in any form) allowed. You may use the given Unit Circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Evaluate trigonometric, exponential, and logarithmic expressions for standard values.
- Good Luck!

Score

1	/12
2	/12
3	/10
4	/24
5	/10
6	/10
7	/24
Total	/100

Formulas that you may or may not find helpful

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\, dx = \ln|\csc x - \cot x| + C$$

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{x\sqrt{x^2-1}}$$

1. (12 pts). ***Simplify*** and find the ***exact*** values of the following expressions.

(a). $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$

(b). $\log_{10} 25 + \log_{10} 4$ $= \log_{10}(25 \cdot 4) = \log_{10}(100) = \boxed{2}$

$10^{?} = 100$

(c). $e^{\ln(\ln(1/e^3))}$ $= \ln\left(\frac{1}{e^3}\right)$

$= \ln(e^{-3})$

$= \boxed{-3}$

2. (12 pts). Given $f(x) = x + x^2 + e^x$, find $(f^{-1})'(1)$. [Note: f is one-to-one. Use the formula for $(f^{-1})'(a)$.]]

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \boxed{\frac{1}{2}}$$

$f^{-1}(1) = ? \Leftrightarrow f(?) = 1$

$x + x^2 + e^x = 1$

By observation, $x = 0$ works

ie $f(0) = 1$

$f^{-1}(1) = 0$

$f'(x) = 1 + 2x + e^x$

$f'(0) = 1 + 2(0) + e^0$

$= 1 + 1$

$= 2$

3. (10 pts). Strontium-90 decays according to the model $m(t) = m_0 e^{kt}$ where m is the mass in mg and t is time in days. The half-life of Strontium-90 is 28 days.
[You do not need a calculator. Leave you answers exact and you do <u>not</u> need to simplify.]

(a). Find the proportionality constant k.

$m(28) = \cancel{m_0} e^{k \cdot 28} = \frac{1}{2} \cancel{m_0}$

$e^{28k} = \frac{1}{2}$

$\ln e^{28k} = \ln(\frac{1}{2})$

$28k = \ln(\frac{1}{2})$

$$\boxed{k = \frac{\ln(\frac{1}{2})}{28} \quad \text{or} \quad k = \frac{-\ln 2}{28}}$$

(b). If a sample has an initial mass of 40 mg, how long will it take to decay to a mass of 8 mg?

$m_0 = 40$

$m(t) = 40e^{kt} = 8$

$e^{kt} = \frac{8}{40} = \frac{1}{5}$

$\ln e^{kt} = \ln(\frac{1}{5})$

$kt = \ln(1/5)$

$$t = \frac{\ln(\frac{1}{5})}{k} = \boxed{\frac{\ln(1/5)}{\left(\frac{\ln(1/2)}{28}\right)}} = \frac{-\ln 5}{\left(\frac{-\ln 2}{28}\right)} = \frac{28 \ln 5}{\ln 2}$$

4. (24 pts). Differentiate the following functions. [*Do* *not* *simplify.*]

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(a). $s(t) = e^{t\cos t\pi} + 5^{8t}$ typo Correction: $\underbrace{e^{t\cos t} + 5^{8t}}$

$$s'(t) = e^{t\cos t} \cdot \frac{d}{dt}[t\cos t] + 5^{8t} \cdot \ln 5 \cdot \frac{d}{dt}[8t]$$

$$= \boxed{e^{t\cos t}\left[t(-\sin t) + \cos t \cdot 1\right] + 5^{8t} \cdot \ln 5 \cdot 8}$$

$$= e^{t\cos t}\left[-t\sin t + \cos t\right] + 5^{8t} \cdot 8\ln 5$$

(b). $y = \sec^{-1}(4x^2)$

$$y' = \frac{1}{4x^2\sqrt{(4x^2)^2 - 1}} \cdot \frac{d}{dx}[4x^2]$$

$$= \boxed{\frac{1}{4x^2\sqrt{16x^4 - 1}} \cdot 8x} = \frac{2}{x\sqrt{16x^4 - 1}}$$

(c). $y = \cosh(\sqrt{x})$

$$y' = \sinh(\sqrt{x}) \cdot \frac{d}{dx}[x^{1/2}]$$

$$= \boxed{\sinh(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}}$$

$$= \frac{\sinh\sqrt{x}}{2\sqrt{x}}$$

5. (10 pts). Find the equation of the tangent line to $y = \ln(x^2)$ at $x = 1$. [Simplify all values.]

① pt. $y = \ln(1)^2 = \ln 1 = 0$

$\Rightarrow$ pt. $(1,0)$

② Slope: $y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$m = y'\big|_{x=1} = \frac{2}{1} = 2$

$\boxed{y - 0 = 2(x-1)}$

6. (10 pts). Find y' in terms of x only for

$y = (\sin x)^x$

$\ln y = \ln(\sin x)^x$

$\ln y = x \cdot \ln(\sin x)$

$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \cdot \ln(\sin x)]$

$\frac{1}{y} y' = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 1$

$y' = (x \cot x + \ln(\sin x)) \cdot y$

$\boxed{y' = (x \cot x + \ln(\sin x)) \cdot (\sin x)^x}$

7. (24 pts). Evaluate the following integrals.

(a). $\displaystyle\int \frac{1}{at+b}\,dt$ where a and b are constants.

$u = at+b$

$du = a\,dt$

$\frac{1}{a}du = dt$

$= \frac{1}{a}\int \frac{1}{u}\,du$

$= \frac{1}{a}\ln|u| + C = \boxed{\frac{1}{a}\ln|at+b| + C}$

(b). $\displaystyle\int \frac{e^{-3x}}{(1+e^{-3x})^2}\,dx$

$u = 1+e^{-3x}$

$du = -3e^{-3x}\,dx$ ✓

$-\frac{1}{3}du = e^{-3x}dx$

$= -\frac{1}{3}\int \frac{1}{u^2}\,du = -\frac{1}{3}\int u^{-2}\,du = -\frac{1}{3}\frac{u^{-1}}{-1} + C = \frac{1}{3u} + C$

$= \boxed{\frac{1}{3(1+e^{-3x})} + C}$

(c). $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$= \int \sin(\ln x)\cdot\frac{1}{x}\,dx$

$= \int \sin u\,du$

$= -\cos u + C = \boxed{-\cos(\ln x) + C}$