Name: _____ **Exam 1**

Math 152, Calculus II – Crawford **18 September 2018**

- No calculators, books, or notes (in any form) allowed. You may use the given Unit Circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Evaluate trigonometric, exponential, and logarithmic expressions for standard values.
- Good Luck!

Score	
1	/12
2	/12
3	/10
4	/24
5	/10
6	/10
7	/24
Total	/100

Formulas that you may or may not find helpful

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{x\sqrt{x^2-1}}$$

1. (12 pts). ***Simplify*** and find the ***exact*** values of the following expressions.

(a). $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

(b). $\log_{10} 25 + \log_{10} 4$

(c). $e^{\ln(\ln(1/e^3))}$

2. (12 pts). Given $f(x) = x + x^2 + e^x$, find $\left(f^{-1}\right)'(1)$. [Note: f is one-to-one. Use the formula for $\left(f^{-1}\right)'(a)$.]

3. (10 pts). Strontium-90 decays according to the model $m(t) = m_0 e^{kt}$ where m is the mass in mg and t is time in days. The half-life of Strontium-90 is 28 days.
[You do not need a calculator. Leave you answers exact and you do not need to simplify.]

(a). Find the proportionality constant k.

(b). If a sample has an initial mass of 40 mg, how long will it take to decay to a mass of 8 mg?

4. (24 pts). Differentiate the following functions. [***Do <u>not</u> simplify.***]

(a). $s(t) = e^{t\cos t} + 5^{8t}$

(b). $y = \sec^{-1}\left(4x^2\right)$

(c). $y = \cosh(\sqrt{x})$

5. (10 pts). Find the equation of the tangent line to $y = \ln(x^2)$ at $x = 1$. [Simplify all values.]

6. (10 pts). Use Logarithmic Differentiation to find y' in terms of x only for

$y = (\sin x)^x$

7. (24 pts). Evaluate the following integrals.

(a). $\displaystyle\int \frac{1}{at+b}\,dt$ where a and b are constants.

(b). $\displaystyle\int \frac{e^{-3x}}{(1+e^{-3x})^2}\,dx$

(c). $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$