The following expansions may be helpful:

$(x+h)^2 = x^2 + 2xh + h^2$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$(x+h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

For each of the following functions, use the limit definition $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to compute the derivative $f'(x)$. Show all your work.

1. $f(x) = x^2$

2. $f(x) = x^3$

3. $f(x) = x^4$

4. $f(x) = x^5$

Summarize your results here:

$f(x) = x^2 \Rightarrow f'(x) =$ __________
$f(x) = x^3 \Rightarrow f'(x) =$ __________
$f(x) = x^4 \Rightarrow f'(x) =$ __________
$f(x) = x^5 \Rightarrow f'(x) =$ __________

5. You should see a pattern. Based on the work above, make an educated guess for the derivative of the following two functions:

(a) $f(x) = x^{56}$

(b) $f(x) = x^n$ where n is any positive integer.