

The Final Exam will cover material from the entire semester.
Use the old review sheets, exams, and quizzes to study previous material.

1. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a). $\int_0^2 t^2 \sqrt{1+t^3} dt = \frac{52}{9}$ *u*-substitution

(b). $\int \sin x \cos(\cos x) dx = -\sin(\cos x) + C$ *u*-substitution

(c). $\int x(3x^2 - 2x)^2 dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 + x^4 + C$ expand/simplify

(d). $\int \sin x \cos x dx = \frac{1}{2} \sin^2 x + C$ OR $-\frac{1}{2} \cos^2 x + C$ *u*-substitution

(e). $\int_0^1 (2-x)^6 dx = \frac{127}{7}$ *u*-substitution

2. Sketch the region bounded by the graphs of the following functions. Find the area of the region.

(a). $f(x) = 3 - 2x - x^2$, $g(x) = -x + 1$ $\frac{9}{2}$

(b). $x = y^2$, $x = -y$ $\frac{1}{6}$

3. Set up, but do not evaluate the integral(s) to find volume of the solid generated by rotating the region bounded by the given curves about the given line.

(a). $y = x^2$, $y = 4x - x^2$ about the line $y = 6$ $V = \int_0^2 \pi [(6 - x^2)^2 - (6 - 4x + x^2)^2] dx$

(b). $xy = 6$, $y = 2$, $y = 6$, $x = 6$ about the line $x = 6$. $V = \int_2^6 \pi \left(6 - \frac{6}{y}\right)^2 dy$

4. The force exerted by gravity on an object sent into space is given by $F(x) = \frac{4.8 \times 10^{11}}{x^2}$ pounds where x is measured in miles from the *center* of the earth. How much work is done to propel a satellite module to 800 miles above the earth. Use 4000 miles for the radius of the earth. [Similar problems not requiring a calculator may be on the test.]

$$W = \int_{4000}^{4800} 4.8 \times 10^{11} x^{-2} dx = 2 \times 10^7 \text{ mile} \cdot \text{lbs} = 1.056 \times 10^{11} \text{ foot} \cdot \text{lbs}$$

5. If 18 J of work is required to stretch a spring 40 cm from its natural length, find the work required to stretch it an additional 30 cm.

Convert units to m. $k = 225 \Rightarrow W = 37.125 \text{ J}$

6. Given $f(x) = \frac{4x^2 + 4}{x^2}$

(a). Find the average value of $f(x)$ on the interval $[-3, -1]$. $f_{ave} = \frac{16}{3}$

(b). Use the Mean Value Theorem for integrals to find all values $x = c$ where $f(c) = f_{ave}$. $x = -\sqrt{3}$

7. Find the value of k so that the average value of $f(x) = kx^2 - x$ on $[0, 2]$ is equal to 4. $k = \frac{15}{4}$

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