INFORMATION FROM THE FUNCTION $y = f(x)$:

1. Domain:
Find the domain of the function by determining all values of x for which the function $f(x)$ is defined.

2. Intercepts:

(a). *x-intercept(s)*: Set $y = 0$ and solve for x. [May have more than one]

(b). *y-intercept*: Set $x = 0$ and find the y-value, i.e. $f(0)$. [Only one]

3. Asymptotes:

(a). *Vertical asymptotes*: If the $\lim_{x \to a} f(x) = \frac{\text{nonzero number}}{0} = +\infty$ or $-\infty$, then a line of the form $x = a$ is a vertical asymptote. [Note: The function is undefined at $x = a$.]

(b). *Horizontal asymptotes*: If the $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$, then a line of the form $y = L$ is a horizontal asymptote.

(c). *Slant asymptotes*: Slant asymptotes are lines of the form $y = mx + b$ that occur in rational functions $f(x) = \frac{p(x)}{q(x)}$ where the degree of the numerator is one higher than the degree of the denominator. Note in this case, the function will be growing or decreasing without bound (i.e. $\lim_{x \to \pm\infty} f(x) = +\infty$ or $-\infty$).

(i) Use long division to rewrite the function as $f(x) = mx + b + \frac{remainder}{denominator}$.

(ii) Verify that $\lim_{x \to \pm\infty} \frac{remainder}{denominator} = 0$.

(iii) Hence, as $x \to \pm\infty$, the function looks like the line $y = mx + b$ which is the slant asymptote.

INFORMATION FROM THE FIRST DERIVATIVE $f'(x)$:

4. Intervals of Increase or Decrease:
To find intervals where the function is increasing or decreasing,

(i). Find the *critical numbers* (i.e. x-values where $f'(x) = 0$ or $f'(x)$ is undefined).

(ii). Draw a number line and separate it into intervals using the critical numbers and asymptotes.

(iii). Choose a number from each interval and find the value of f' at that number.

(iv). If f' is negative, then f is decreasing on that interval.
If f' is positive, then f is increasing on that interval.

5. Local Maximum and Minimum Values:

(a). A *local maximum* occurs at any number x in the domain where f is increasing on the left of that number and decreasing on the right of that number.

(b). A *local minimum* occurs at any number x in the domain where f is decreasing on the left of that number and increasing on the right of that number.

(c). Plug these $x-$numbers into the original $f(x)$, to find the *local minimum and maximum values* of f.

INFORMATION FROM THE SECOND DERIVATIVE $f''(x)$:

6. Intervals of Concave Up or Concave Down:
To find intervals where the function is concave up or concave down,

(i). Find the interesting numbers (i.e. x-values where $f''(x) = 0$ or $f''(x)$ is undefined).

(ii). Draw a number line and separate it into intervals using these interesting numbers and asymptotes.

(iii). Choose a number from each interval and find the value of f'' at that number.

(iv). If f'' is negative, then f is concave down on that interval.
If f'' is positive, then f is concave up on that interval.

7. Inflection Points:

(a). A *inflection point* occurs at any number x in the domain where f is concave up on the left of that number and concave down on the right side of that number or vice versa.

(b). Plug these $x-$numbers into the original $f(x)$, to find the actual *inflection point* of f.

FINALLY:

8. Sketch the Curve:
Use all the information obtained above to sketch the curve:

(i). Draw asymptotes from part **(3)** with dashed lines.

(ii). Plot the intercepts, maximum and minimum values, and inflection points found in **(2)**, **(5)**, & **(7)**.

(iii). Sketch a curve that passes through these points that increases and decreases as found in part **(4)**, with concavitiy as found in part **(6)**, and approaches the asymptotes from part **(3)**.