1. For each pair of functions f and g, find the composite function $F = f \circ g$. [You do not need to simplify.]

(a). $f(x) = x^4 + x^2$, $g(x) = 3x - 2x^5$

(b). $f(x) = x^8$, $g(x) = \dfrac{\sin x}{x-1}$

(c). $f(x) = \sqrt{x}$, $g(x) = \tan x$

(d). $f(x) = \tan x$, $g(x) = \sqrt{x}$

2. For each function F below, find a pair of functions f and g such that $F = f \circ g$.

(a). $F(x) = (6x^2 - 2x + 3)^2 - 4$

(b). $F(x) = \left(\dfrac{5-3x}{x+2}\right)^9$

(c). $F(x) = 3\sin x - \sqrt{\sin x}$

(d). $F(x) = \tan(\pi x + 1)$

3. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in inches) of the outer circle is given by $r(t) = 10t$, where t is the time (in seconds) after the pebble strikes the water. The area of a circle is given by the function $A(r) = \pi r^2$.

(a). Find $(A \circ r)(t) = A(r(t))$

(b). Fill in the blank to explain in words what it means $A \circ r$ means: The expression from part (a)

$A =$ ____________ gives the ____________ of the outer circle as a function of ____________ .

4. Let $f(x) = x^2 + 5x - 3$ and $g(x) = 3x^2 + 2x$.

(a). Find $F(x) = (f \circ g)(x)$. <u>Simplify/expand</u> your answer.

(b). Find $F'(x)$.

(c). Find $f'(x)$ and $g'(x)$.

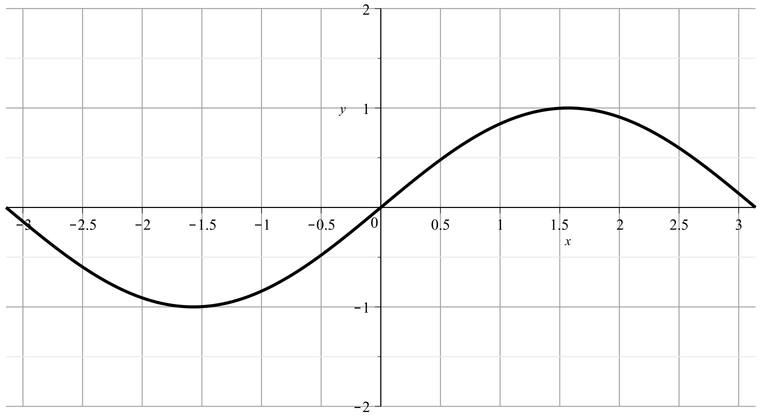
(d). Find $f'(g(x))$ [i.e. the composition of $f' \circ g$].

(e). Find $f'(g(x)) \cdot g'(x)$ and <u>simplify</u> your answer.

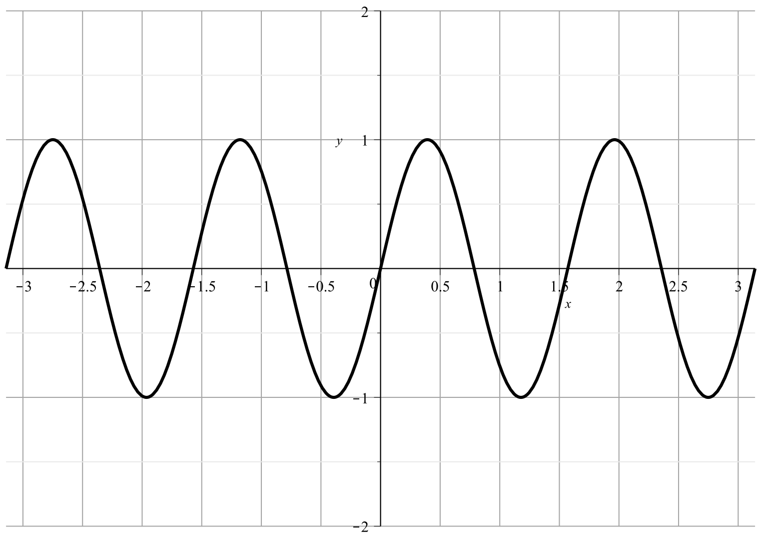
(f). Compare the result of part **(b)** with part **(e)**:

TRUE or FALSE: $F'(x) = f'(g(x)) \cdot g'(x)$

Graph of $f(x) = \sin(x)$:



Graph of $F(x) = \sin(4x)$:



5. Use the graphs above to help answer the following questions.

(a). How many completes cycles does $f(x) = \sin(x)$ make in the interval $[-\pi, \pi]$? ______________

(b). How many completes cycles does $f(x) = \sin(4x)$ make in the interval $[-\pi, \pi]$? ______________

(c). So $F(x) = \sin(4x)$ is ***changing at a rate*** that is ______________ times as fast as the ***rate*** $f(x) = \sin(x)$ ***changes***.

(d). Since the ________________ represents the ***rate of change***, we expect the derivative of $F(x) = \sin(4x)$ to be ______________ times as large as the derivative of $f(x) = \sin(x)$.

(e). Sketch the tangent line to $f(x) = \sin(x)$ at $x = 0$. Estimate the slope of this tangent line: ______________

(f). Sketch the tangent line to $f(x) = \sin(4x)$ at $x = 0$. Estimate the slope of this tangent line: ______________

(g). Do your answers to parts **(e)** and **(f)** confirm your guess in part **(d)**?

6. Suppose the graphs on the next page are given for a car company where

$c(w)$ = number of cars produced by w workers and

$p(c)$ = profit in dollars from producing c cars.

(a). Let $P(w) = (p \circ c)(w) = p(c(w))$ = profit from w workers.

(***i***) If there are $w = 200$ workers, how many cars c are produced?

(***ii***) If c is the number of cars found in part (***i***), what is the profit p?

(***iii***) Use parts (***i***) and (***ii***) to determine the profit P when you have 200 workers, i.e. find $P(200) = (p \circ c)(200) = p(c(200))$.

(***iv***) Repeat parts (***i***)-(***iii***) to find $P(0), P(100)$ and $P(300)$.

(***v***) Use the results of (***iii***)-(***iv***) to complete the following table.

w	$P(w)$
0	
100	
200	
300	

(***vi***) Use the table to sketch the graph for $P(w) = (p \circ c)(w) = p(c(w))$ on following page.

(b). If $c'(200) = 10$, then the slope of the tangent line at $w = 200$ is _________ . So if 200 workers are currently working, approximately how many more cars will be produced by adding one more worker?

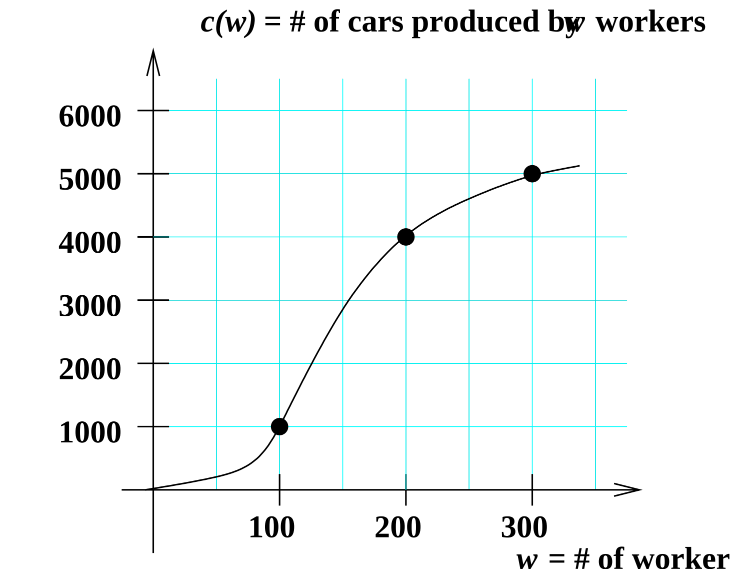
(c). If $p'(4000) = 450$, then the slope of the tangent line at $c = 4000$ is __________ . So if 4000 cars are currently being produced, approximately how much more profit will be made by producing one more car?

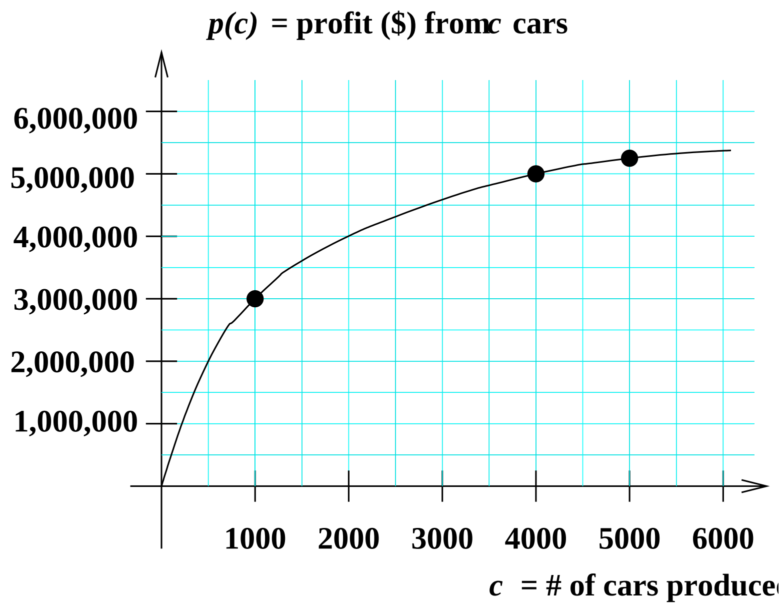
(d). Based on your answers to parts (b) and (c), fill in the following blanks:
If 200 workers are currently working and you add more workers, it will result in _________ more cars per worker and ___________ profit per car. So the overall increase to profit is [Fill in the correct numbers below.]

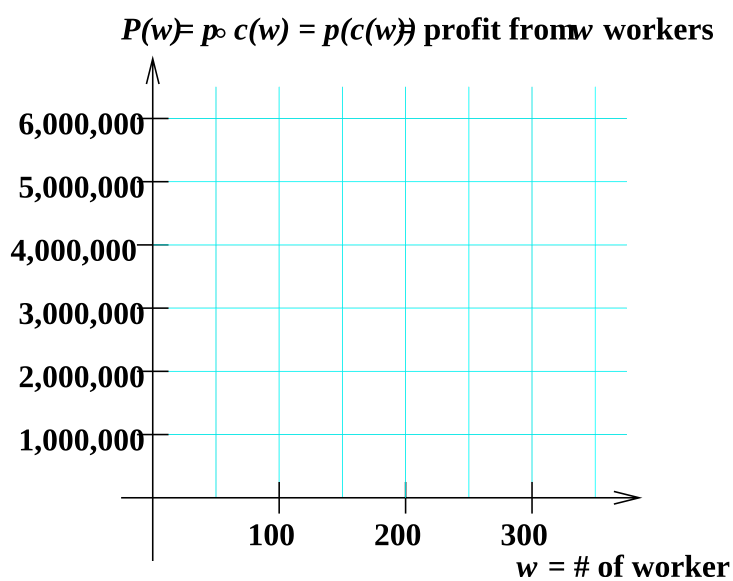
$$\_\_\_\_\_\_\_\_\_ \ \frac{\text{cars}}{\text{worker}} \times \ \_\_\_\_\_\_\_\_\_\_ \ \frac{\text{profit(\$)}}{\text{cars}} = \ \_\_\_\_\_\_\_\_\_\_\_ \ \frac{\text{profit(\$)}}{\text{worker}}$$

(e). Based on your answer to part (d), $P'(200) =$ ___________.
[i.e. The change in profit $P(w)$ from adding one more worker to the current 200 working.]

(f). Use your answers from (d) and (e) to write a relationship between $P'(200), c'(200)$, and $p'(4000)$.


c(w) = # of cars produced by w workers
6000
5000
4000
3000
2000
1000
100
200
300
w = # of worker


p(c) = profit ($) from c cars
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
1000
2000
3000
4000
5000
6000
c = # of cars produce


P(w) = p∘ c(w) = p(c(w)) = profit from w workers
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
100
200
300
w = # of worker