Recall the TANGENT LINE PROBLEM: Given a curve $y = f(x)$ and a point $P(a, f(a))$ on the curve, find the equation for the tangent line to the curve $y = f(x)$ at the point P.

[Sketch]

In order to write the equation of a line, we need

(1). Point: $\surd$ $P(a, f(a))$ is given

(2). Slope: Not given. Also not given 2 points on the tangent line (we only have 1 point), so we can't compute the slope.

$\Rightarrow$ Approximate the slope of the tangent line by finding the slope of the secant line through P and another arbitrary point Q on the curve.

If x is the x-coordinate of point Q, then the y-coordinate is $\underline{\quad f(x) \quad}$. ie. $Q(x, f(x))$

So the slope of the secant line through $P(a, f(a))$ and $Q(x, f(x))$ is given by

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x) - f(a)}{x - a}$$

The approximated slope m_{PQ} will get closer to the slope m of the tangent line as:

Q gets closer to P (i.e. $Q \to P$)

$\Rightarrow x$ gets closer to a (i.e. $x \to a$)

i.e. $\dfrac{f(x) - f(a)}{x - a} = m_{PQ} \to m$ as $x \to a$ This is a LIMIT!

(In fact, this limit is how we define the tangent line)

<u>**Def**</u>. The <u>TANGENT LINE</u> to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with the slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Ex Given $f(x) = x^2$ at $x = 2$

(a). Approximate the slope of the tangent line at $x = 2$ by making a table of values for the slope of the secant line m_{PQ} near $x = 2$.

P: At $x = 2$, $y = f(2) = 2^2 = 4$ $\Rightarrow$ $P(2, 4)$

Q: Any point on $y = x^2$ $\Rightarrow$ $Q(x, x^2)$

So $m_{PQ} = \dfrac{f(x) - f(2)}{x - 2} = \dfrac{x^2 - 4}{x - 2}$

x	1.5	1.75	1.9	1.99	2	2.01	2.1	2.5	2.75
$m_{PQ} = \dfrac{x^2 - 4}{x - 2}$	3.50	3.75	3.90	3.99	??	4.01	4.1	4.25	4.5

From the table of values, guess the slope of the tangent line to be $m =$ ___4___.

i.e. $\lim_{x \to 2} m_{PQ} = m =$ ___4___

(b). Verify your guess in part (a) by calculating the limit analytically.

$$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \to 2} x + 2 = 4$$

(c). Find the equation of the tangent line to $f(x) = x^2$ at $x = 2$ $y - 4 = 4(x - 2)$

(d). Sketch the tangent line from part (c) on the graph of $f(x) = x^2$ below. Does this line appear to be the tangent line (just barely touching) at P?