Basic Limits

1. $\lim_{x \to a} c =$

2. $\lim_{x \to a} x =$

Limit Laws Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and c is a constant, then

3. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

4. $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

5. $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

6. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, if $\lim_{x \to a} g(x) \neq 0$

7. $\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n$ for positive integer n

Even More Special Limits and Laws

8. $\lim\limits_{x\to a} x^n = a^n$ for positive integer n

9. $\lim\limits_{x\to a} x^{1/n} = \lim\limits_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ for positive integer n and if n is even, $a \geq 0$

10. $\lim\limits_{x\to a}[f(x)]^{1/n} = \lim\limits_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)}$ for positive integer n. [In the case that n is even, $f(x) \geq 0$]

Ex: Evaluate the following limit, justifying each step with limit laws.

$$\lim_{x\to 2} \frac{3x^2+2x+2}{\sqrt{2x-1}} = \frac{\lim\limits_{x\to 2}(3x^2+2x+2)}{\lim\limits_{x\to 2}\sqrt{2x-1}} = \frac{\lim\limits_{x\to 2}(3x^2+2x+2)}{\sqrt{\lim\limits_{x\to 2}(2x-1)}}$$ by Law

$$= \frac{\lim\limits_{x\to 2} 3x^2 + \lim\limits_{x\to 2} 2x + \lim\limits_{x\to 2} 2}{\sqrt{\lim\limits_{x\to 2} 2x - \lim\limits_{x\to 2} 1}}$$ by Law

$$= \frac{3\lim\limits_{x\to 2} x^2 + 2\lim\limits_{x\to 2} x + \lim\limits_{x\to 2} 2}{\sqrt{2\lim\limits_{x\to 2} x - \lim\limits_{x\to 2} 1}}$$ by Law

$$=$$ by Law

To find $\lim_{x \to a} f(x)$, we can use ______________________________

when the function is "nice" at $x = a$.

Strategy for Finding Limits $\left[\text{i.e. Evaluating } \lim_{x \to a} f(x) \text{ analytically}\right]$

1. If you get

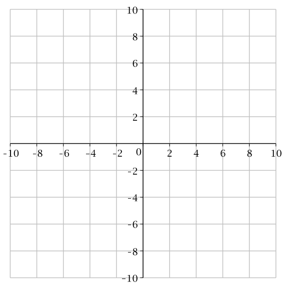
2. If you get

3. If you get

Ex $\lim_{x\to 4} \frac{x^2-16}{x-4}$ Note: $f(x) = \frac{x^2-16}{x-4}$

(a). What is the domain of $f(x)$?

(b). Use your calculator to graph the function and sketch it below. Is the graph what you expected?



(c). From your graph, determine $\lim_{x\to 4} \frac{x^2-16}{x-4}$

(d). Observe:

$f(x) = \frac{x^2-16}{x-4}$