1. Suppose you have a friend that lives 90 miles away in Milwaukee and that it takes you 1 hour and 40 minutes to get to her house.

(a). What is your average speed in miles per hour (mph)?

(b). At any point in time were you going 20 mph? If so, explain how you know this.

What is Calculus?

Question 1(a) above is a typical Pre-calculus/algebra question: What is the ***average*** speed?

The corresponding Calculus question would be:

Question 1(b) above is another typical Calculus question. The reason you knew that you must have driven 20 mph is based on the idea that the velocity must be a ________________ function.

Another Calculus question might be the follow-up question: At what time were you going 20 mph?

2. Suppose we just stopped at the last toll booth and there are clear roads. The following data gives the total distance traveled after we start moving again.

time (sec)	0	1	2	3	4	5
distance (ft)	0	8	32	73	130	203

Calculus Question : Find the *instantaneous* velocity at exactly $t = 1$ second.

But since we don't know how to answer this yet, we're going to approximate it using techniques we already know. $\Rightarrow$ Use *average* velocity.

(a). What is the average velocity over the interval $1 \leq t \leq 5$ seconds?

$v_{avg} =$

(b). Complete the table below by computing the average velocity for shorter time intervals.

time interval	Δt (sec)	average velocity (ft/sec)
$1 \leq t \leq 5$	4	48.75
$1 \leq t \leq 4$	3	
$1 \leq t \leq 3$	2	
$1 \leq t \leq 2$	1	

(c). Which velocity do you think is the best estimate of your speed at the exact instant when $t = 1$? Why?

(d). Suppose we have more detailed data in tenths of seconds:

time (sec)	1.1	1.2	1.3	1.4	1.5
distance (ft)	9.81	11.66	13.68	15.91	18.55

Complete the table below by computing the average velocity for even shorter time intervals.

time interval	Δt (sec)	average velocity (ft/sec)
$1 \leq t \leq 1.5$	0.5	21.10
$1 \leq t \leq 1.4$	0.4	
$1 \leq t \leq 1.3$	0.3	
$1 \leq t \leq 1.2$	0.2	
$1 \leq t \leq 1.1$	0.1	

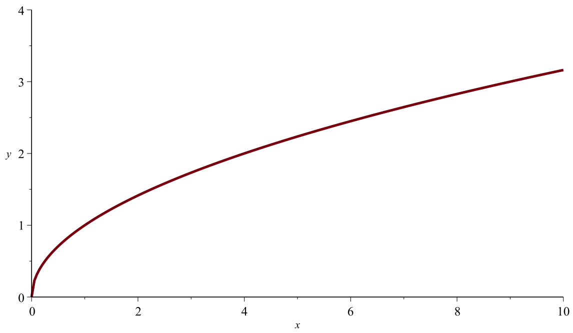
(e). Do the values for the average velocity seem to be getting closer to a specific value? If so, what value?

This problem illustrates using a limit (________________________________) to find the value at one instant.

Informal Description of a <u>Tangent Line</u> (Not the Formal Definition):

<u>The Tangent Line Problem</u>:

3. Sketch the graph of $y = \sqrt{x}$ below and label the point $P(4, 2)$. Draw the tangent line to the curve at P.



(a). Let Q be the point corresponding to $x = 9$. Find the slope of the secant line PQ. $m_{PQ} =$

(b). Let Q be any point (other than P) on the curve, i.e. $Q(x, \quad)$. Find an expression (involving x) for the slope of the secant line PQ.

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

(c). Complete the table below to find the slope of the secant line PQ for the given x-coordinates of Q. Use the expression from part (b) for computing the slope. [Keep 6 decimal places.]

x	$m_{PQ} =$
1.0	
3.0	
3.5	0.258343
3.9	0.251582
3.99	0.250156
3.999	0.250016
4	??
4.001	0.249984
4.01	0.249844
4.1	0.248457
4.5	0.242641
5.0	0.236068
6.0	0.224745
9.0	

(d). Why can't we use the slope formula to compute a slope when the x-coordinate of Q is 4 (i.e. when P and Q are the same point)?

(e). Based on the table above, guess the value of the slope of the tangent line at $P(4, 2)$.

(f). Use this value to write an equation of the tangent line.

HW: Section 1.4, p. 49: #5, 6, 7, 1, 3 [Note: Estimated values may differ from answers in the back of the book, but should be close.]