Math 151 **31 October 2019**

Books, notes (in any form), calculators, etc., are not allowed. You must *show all your work* for full credit. Good Luck!

1. (15 pts) Given the following function and its derivatives

$$f(x) = \frac{x^2-4}{2x^2-2} = \frac{x^2-4}{2(x^2-1)} \qquad f'(x) = \frac{3x}{(x^2-1)^2} \qquad f''(x) = \frac{-3(3x^2+1)}{(x^2-1)^3}$$

(a). Fill in the following information about the function and its graph. Show all work and write "none", if applicable.

domain:	All Real numbers, except $x = \pm 1$
x-intercept(s):	$(\pm 2, 0)$
y-intercept:	$(0, 2)$
vertical asymptote(s):	$x = \pm 1$
horizontal asymptote(s):	$y = \frac{1}{2}$
~~slant asymptote:~~	~~N/A~~
critical numbers:	$x = 0$
intervals where increasing:	$(0,1) \cup (1,\infty)$
intervals where decreasing:	$(-\infty,-1) \cup (-1,0)$
coordinates of local max(s):	None
coordinates of local min(s):	$(0, 2)$
intervals where concave up:	$(-1, 1)$
intervals where concave down:	$(-\infty,-1) \cup (1,\infty)$
Inflection Point(s):	None

Domain: $2(x^2-1) \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow x \neq \pm 1$

x-int ($y=0$): $x^2-4=0 \Rightarrow x^2=4 \Rightarrow x = \pm 2$

y-int ($x=0$): $f(0) = \frac{0^2-4}{2(0)^2-2} = \frac{-4}{-2} = 2 \Rightarrow (0,2)$ pt

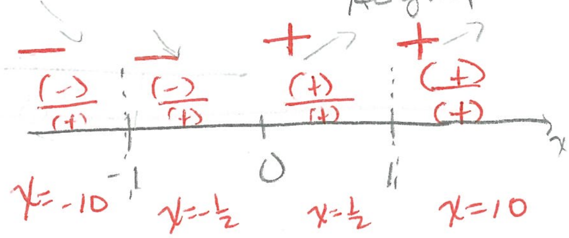
V.A.: $2(x^2-1) = 0 \Rightarrow x = \pm 1$

H.A.: $\lim_{x\to\infty} \frac{x^2-4}{2x^2-2} = \frac{1}{2}$; $\lim_{x\to-\infty} \frac{x^2-4}{2x^2-2} = \frac{1}{2}$

$f' = 0$: $3x = 0$, $x = 0$

f' DNE: $(x^2-1)^2 = 0$, $x = \pm 1$ Asymptotes

$f' = \frac{3x}{(x^2-1)^2}$

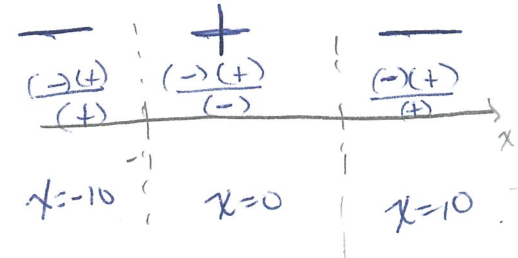


Local min @ $x=0$; $f(0) = 2$ (work above)

$f'' = 0$: $-3(3x^2+1) = 0$, $3x^2+1 = 0$ Not possible

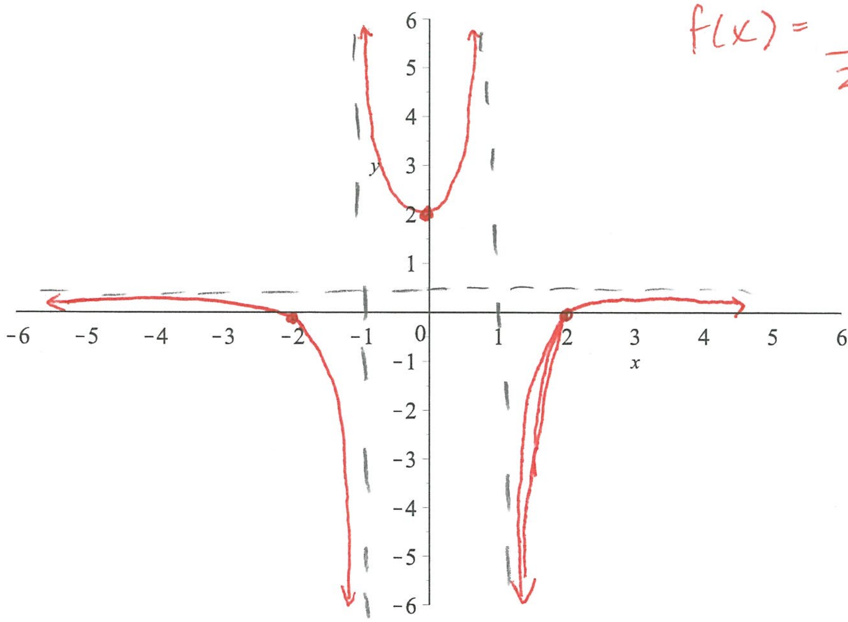
f'' DNE: $(x^2-1)^3 = 0$, $x = \pm 1$ Asymptotes

$f'' = \frac{-3(3x^2+1)}{(x^2-1)^3}$



(b). Sketch the graph of the function on the set of axes provided. Label any maximum and minimum values and inflection points.

$$f(x) = \frac{x^2-4}{2x^2-2}$$



Just an extra set of axes, in case you need it.