Name: Key

Math 151 Calculus I – Crawford

Quiz 2

11 October 2019

Books and notes (in any form) are not allowed. You may use the given calculator. *But you must show your set up and work for full credit.* Good Luck!

1. (5 pts) Given the implicity defined curve $\cos y = y + xy^2 + x$,

(a). Find the derivative y'.

$$\frac{d}{dx}\left[\cos y = y + xy^2 + x\right]$$

$$-\sin y \cdot y' = y' + x \cdot 2yy' + y^2 \cdot 1 + 1$$

$$-\sin y \cdot y' - y' - 2xyy' = y^2 + 1$$

$$y'(-\sin y - 1 - 2xy) = y^2 + 1$$

$$\boxed{y' = \frac{y^2+1}{-\sin y - 1 - 2xy}} = -\frac{y^2+1}{\sin y + 1 + 2xy}$$

(b). Find an equation for the tangent line to the ~~following~~ given implicitly defined curve at the point $(1, 0)$.

① pt ✓ $(1, 0)$

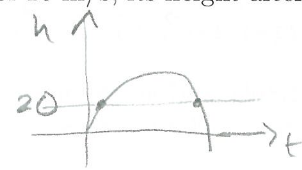
② Slope: $m = y'\Big|_{(1,0)} = \frac{0^2+1}{-\underbrace{\sin(0)}_{0} - 1 - \underbrace{2(1)(0)}_{0}}$

$$= \frac{1}{-1} = -1$$

$$\boxed{y - 0 = -1(x-1)} \Rightarrow y = -x + 1$$

2. (5 pts) If a rock is thrown vertically upward from the surface of Mars with a velocity of 16 m/s, its height after t seconds is given by

$h(t) = 16t - 1.86t^2$



(a). What is the velocity of the rock after 2 s?

$v(t) = h'(t) = 16 - 2(1.86)t = 16 - 3.72t$

$v(2) = 16 - 3.72(2) = \boxed{8.56 \text{ m/s}}$

(b). What is the velocity of the rock when its height is 20 m on its way down?

$h = 20$

$\Rightarrow 16t - 1.86t^2 = 20$

$1.86t^2 - 16t + 20 = 0$

$t = \dfrac{+16 \pm \sqrt{(-16)^2 - 4(1.86)(20)}}{2(1.86)}$

$t = \dfrac{16 \pm \sqrt{107.2}}{3.72}$

≈ 1.5178 or $\underbrace{7.0843}_{\text{way down}}$

$v(7.0843) = 16 - 3.72(7.0843)$

$\approx \boxed{-10.35 \text{ m/s}}$

3. (5 pts) Given $f(x) = \dfrac{1}{(1-x)^3} = (1-x)^{-3}$,

(a). Find the linearization $L(x)$ at $x = 0$.

① pt $f(0) = \dfrac{1}{(1-0)^3} = 1 \Rightarrow (0,1)$

② slope: $f'(x) = -3(1-x)^{-4}\cdot(-1) = \dfrac{3}{(1-x)^4} \Rightarrow f'(0) = \dfrac{3}{(1)^4} = 3$

$L(x) = f(0) + f'(0)(x-0) \Rightarrow \boxed{L(x) = 1 + 3(x-0)} = 1 + 3x$

(b). Use the linearization from part (a) to approximate $\dfrac{1}{(0.99)^3}$. i.e., Use $L(x)$ to approximate $f(0.01)$.

$\dfrac{1}{(0.99)^3} = f(0.01) \approx L(0.01) = 1 + 3(0.01)$

$= \boxed{1.03}$

Note: $\dfrac{1}{(0.99)^3} \approx 1.030610152$

(But not required)