The Final Exam will cover material from the entire semester. This Review Sheet is for material from Exams 1, 2, & 3. Please see the New Material Review Sheet to review the material after Exam 3. Use previous exams and quizzes to also study previous material.

1. Solve the following inequalities. Write your solution in interval notation and sketch it on the numberline.
$2x^2 + x \geq 3$ $(-\infty, -\frac{3}{2}] \bigcup [1, \infty)$

2. Given $f(x) = \begin{cases} -1, & x \leq -1 \\ x, & -1 < x \leq 2 \\ x^2+1, & x > 2 \end{cases}$

(a). Find the domain and sketch the function. domain: All real numbers; sketch

(b). Find $f(-2)$, $f(2)$, and $f(4)$. $f(-2) = -1$, $f(2) = 2$, $f(4) = 17$

3. Section 1.4 #7(a)(i)

4. Section 1.5 #7

5. Evaluate the following limits, if they exist (clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit). If the limit does not exist, **explain the reason why**.

(a). $\lim_{x \to 0} \frac{x-3}{x(x+4)}$ DNE (one-sided limits are different) **(b)**. $\lim_{x \to -4} \frac{x^2+2x-8}{x+4} = -6$ **(c)**. $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$

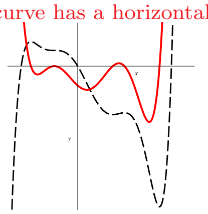
6. For each of the following functions,
(i) find all of the x-values, if any, where g(x) is discontinuous and
(ii) indicate whether it is a removable, infinite, or jump discontinuity.

(a). $g(x) = \frac{x^2 - x - 6}{x(x^2-9)}$ infinite at $x = 0$, infinite $x = -3$, removable at $x = 3$

7. Given the function $f(x) = x^3 - 2x^2 + 8x - 1$, use the Intermediate Value Theorem to show that there is a number c where $0 < c < 2$, such that $f(c) = 6$. $f(0) = -1$ and $f(2) = 15$. Since $-1 \leq 6 \leq 15$ AND f is continuous, then the IVT guarantees $f(x)$ must pass through $y = 6$ for some value of $x = c$ in the interval $(0, 2)$.

8. Given $f(x) = x^2 - x$, ***use the limit definition*** $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find the slope of the tangent line to the curve at $x = 2$. Use your result to write an equation of the tangent line at $x = 2$. $m = 3$, $y - 2 = 3(x - 2)$

9. Both the function f and its derivative f' are plotted on the same set of axes below. Which curve represents the function and which curve represents the derivative? ***Justify your answer.*** dashed (black) is $f(x)$ solid (red) is $f'(x)$ Everywhere the dashed (black) curve has a horizontal tangent line (slope = 0), the solid (red) curve goes through 0.



10. Differentiate the following using ***Differentiation Rules***.

(a). $y = 10x^3 - 3x + 7$ $\quad y' = 30x^2 - 3$

(b). $y = \dfrac{x + 4x^3 - 3}{x^4 + 3}$ $\quad y' = \dfrac{(x^4+3)(1+12x^2) - (x+4x^3-3)(4x^3)}{(x^4+3)^2}$

(c). $g(\theta) = \dfrac{\theta^3 \sin\theta}{2\theta + \sec\theta}$ $\quad \dfrac{g'(\theta) = (2\theta+\sec\theta)(\theta^3\cos\theta + (\sin\theta)3\theta^2) - \theta^3\sin\theta(2+\sec\theta\tan\theta)}{(2+\sec\theta)^2}$

(d). $s(t) = (3t^3 - t^2 + 7)^{23}$ $\quad s'(t) = 23(3t^3 - t^2 + 7)^{22}(9t^2 - 2t)$

(e). $f(\theta) = \theta\sin(\theta^2 + 1)$ $\quad f'(\theta) = 2\theta^2\cos(\theta^2+1) + \sin(\theta^2+1)$

(f). $y = \dfrac{x(2x^4+4)^8}{\tan 2x}$ [Do not simplify!] $\quad \dfrac{dy}{dx} = \dfrac{\tan 2x \cdot [x \cdot 8(2x^4+4)^7 \cdot 8x^3 + (2x^4+4)^8 \cdot 1] - x(2x^4+4)^8(\sec^2(2x)\cdot 2)}{\tan^2 2x}$

11. Find an equation of the tangent line to the curve $y = 3\tan x$ at the point $x = \dfrac{\pi}{3}$ $\quad y - 3\sqrt{3} = 12\left(x - \dfrac{\pi}{3}\right)$

12. Solve the following equations for x.

(a). $2\sin^2 x - \sqrt{2}\sin x = 0$ (x in $[0, 2\pi]$) $\quad x = 0, \pi, 2\pi, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

13. Find the equation of the tangent line to the curve $y = \sqrt[3]{2x^2 - 5}$ at $x = 4$. $\quad y - 3 = \dfrac{16}{27}(x - 4)$

14. Given $f(x) = g(3x^2)$, find f' in terms of g'. $\quad f'(x) = g'(3x^2) \cdot 6x$

15. Given the curve defined by $x^2 + y^2 = 3 + xy$, find $\dfrac{dy}{dx}$ $\quad \dfrac{dy}{dx} = \dfrac{y - 2x}{2y - x}$

16. A tank holds 1000 gallons of water, which drains from the bottom of the tank in 50 minutes. Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V = 1000\left(1 - \dfrac{1}{50}t\right)^2$ for $0 \le t \le 50$. Find the rate at which the water is draining from the tank after 10 minutes. Include units in your answer. -32 gallons/min.

17. If a stone is thrown vertically upward on the moon with a velocity of 8 m/s, its height after t seconds is given by $y = 8t - 0.83t^2$, [Calculator*]

(a). What is the velocity after 2 s? 4.68 m/s

(b). What is the velocity at impact? -8 m/s

18. Any Section 2.7 applications.

19. Given $f(x) = 2\sqrt{x} - x$, find the <u>absolute</u> maximum and minimum <u>*values*</u> of $f(x)$ on the interval $[0, 9]$.
Absolute Max $= 1$ at $x = 1$; Absolute Min $= -3$ at $x = 9$.

20. Given $f(x) = 36x + 3x^2 - 2x^3$

(a). Find the intervals of increase or decrease. Increasing: $(2, 3)$ Decreasing: $(-\infty, 2) \cup (3, \infty)$

(b). Find the local maximum and minimum values. Min $= -44$ at $x = -2$; Max $= 81$ at $x = 3$.

(c). Find the intervals of concavity and the inflection points. Up: $\left(-\infty, \dfrac{1}{2}\right)$ Down: $\left(\dfrac{1}{2}, \infty\right)$ Inf. Pts.: $\left(\dfrac{1}{2}, \dfrac{37}{2}\right)$

21. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\displaystyle\lim_{x \to -\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4} = \frac{2}{3}$

[1] Similar non-calculator problems could be given.

22. Find the horizontal asymptote(s) of the following function. [You may use shortcuts.] $f(x) = \dfrac{3 - x^2 + 4x^3}{x^4 + 2x}$ $y = 0$

23. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-2}$ on the interval [2, 6] and find all values of c that satisfy the MVT. $c = 3$

24. If a box with a square base and open top is to hold 4 ft^3, find the dimensions of the box that will require the least amount of material. $2' \times 2' \times 1'$

25. Section 3.7 #35

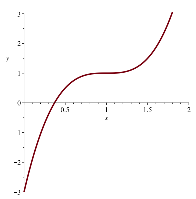
26. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$, [The graph is given below.]

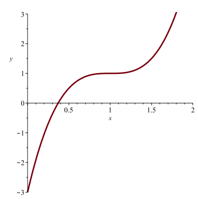
(a). Explicitly write out Newton's formula for finding the root of this function. $x_{n+1} = x_n - \dfrac{4x_n^3 - 12x_n^2 + 12x_n - 3}{12x_n^2 - 24x_n + 12}$

(b). Start with an initial guess of $x_0 = 0.5$ and iterate Newton's Method once. [***Do <u>not</u> simplify your answer!***]
$x_1 = 0.5 - \dfrac{4(0.5)^3 - 12(0.5)^2 + 12(0.5) - 3}{12(0.5)^2 - 24(0.5) + 12}$

(c). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Yes

(d). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?
No, it won't work since the tangent line is horizontal when $x_0 = 1.0$.





27. Find the antiderivatives of $f(x) = \sqrt{x} - \sqrt{3}x^2$ $F(x) = \frac{2}{3}x^{3/2} - \frac{\sqrt{3}}{3}x^3 + C$

28. Given that $g'(\theta) = -\sec^2\theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$. $g(\theta) = -\tan\theta + \sqrt{3}$

29. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5].
$R_5 = \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] = 1 \cdot [f(2) + f(3) + f(4) + f(5) + f(6)] = \boxed{1 \cdot [3/2 + 3/3 + 3/4 + 3/5 + 3/6]} = \frac{87}{20}$

30. Section 4.3: #3

31. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$ for $F(x) = \displaystyle\int_0^x t\cos t\, dt$ $F'(x) = x\cos x$

32. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \le t \le 6$.

(a). Find the displacement 0

(b). Find the total distance traveled $\dfrac{32}{3} + \left|-\dfrac{32}{3}\right| = \dfrac{64}{3}$

33. Evaluate the following integrals. [**Note:** <u>You may or may not need to use substitution</u>.] Check your answer by differentiating the result.

(a). $\displaystyle\int 3x^5 - 4x^3 + 6x + 2\, dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$

(c). $\displaystyle\int_1^3 \frac{x^2+1}{x^2}\, dx = \frac{8}{3}$

(b). $\displaystyle\int (3x-1)(3x^2-2x)^2\, dx = \frac{1}{6}(3x^2-2x)^3 + C$

(d). $\displaystyle\int \theta\sin(3\theta^2)\, d\theta = -\frac{1}{6}\cos(3\theta^2) + C$