

1. Evaluate the following limits. [Show all work - no shortcuts].

(a).  $\lim_{x \rightarrow \infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x} = 0$

(b).  $\lim_{x \rightarrow -\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4} = \frac{2}{3}$

2. Find the horizontal asymptote(s) of the following function. [You may use shortcuts.]

$$f(x) = \frac{2x + 1}{\sqrt{4x^2 - x}}$$

$\lim_{x \rightarrow \infty} f(x) = 1$ ;  $\lim_{x \rightarrow -\infty} f(x) = -1$       HA:  $y = 1$  and  $y = -1$

3. Given the following function and its derivatives

$$f(x) = \frac{x}{9 - x^2} \quad f'(x) = \frac{x^2 + 9}{(9 - x^2)^2} \quad f''(x) = \frac{2x(x^2 + 27)}{(9 - x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain: All real numbers, except $x \neq \pm 3$	slant asymptote: none	coordinates of local max/min(s): none
x-intercept(s): $x = 0$	critical numbers: none	intervals where concave up: $(\infty, -3) \cup (0, 3)$
y-intercept: $y = 0$	intervals where increasing: $(\infty, -3) \cup (-3, 3) \cup (3, \infty)$	intervals where concave down: $(-3, 0) \cup (3, \infty)$
vertical asymptote(s): $x = 3, x = -3$	intervals where decreasing: none	inflection point(s): $(0, 0)$

Use your calculator to graph the function to check and see if you sketched it correctly.

4. If a box with a square base and open top is to hold  $4 \text{ ft}^3$ , find the dimensions of the box that will require the least amount of material.  $2' \times 2' \times 1'$

5. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is  $150\pi$ .   
  $V = 250\pi$     ( $r = 5, h = 10$ )

6. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft.    Rect. portion:  $\frac{16}{\pi+4} \times \frac{32}{\pi+4}$  ft.

7. Given  $f(x) = 4x^3 - 12x^2 + 12x - 3$ , [The graph is given below.]

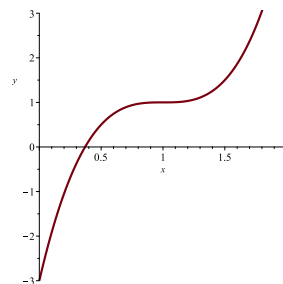
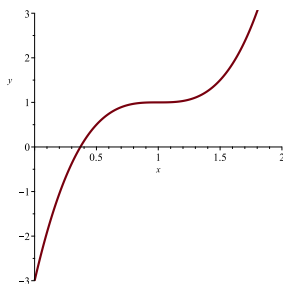
(a). Explicitly write out Newton's formula for finding the root of this function.  $x_{n+1} = x_n - \frac{4x_n^3 - 12x_n^2 + 12x_n - 3}{12x_n^2 - 24x_n + 12}$

(b). Start with an initial guess of  $x_0 = 0.5$  and iterate Newton's Method once. [Do not simplify your answer!]

$$x_1 = 0.5 - \frac{4(0.5)^3 - 12(0.5)^2 + 12(0.5) - 3}{12(0.5)^2 - 24(0.5) + 12}$$

(c). Starting with  $x_0 = 0.5$ , demonstrate Newton's method by marking  $x_0, x_1, x_2, \dots$  and the associated tangent lines on the graph of  $f(x)$ . Does it seem like Newton's method will work if you start with this initial guess? Yes

(d). Starting with  $x_0 = 1.0$ , demonstrate Newton's method by marking  $x_0, x_1, x_2, \dots$  and the associated tangent lines on the graph of  $f(x)$ . Does it seem like Newton's method will work if you start with this initial guess? Why or why not?   
 No, it won't work since the tangent line is horizontal when  $x_0 = 1.0$ .



8. Find the antiderivatives for the following functions.

(a).  $h(x) = 3x^3 - 7x^2$        $H(x) = \frac{3}{4}x^4 - \frac{7}{3}x^3 + C$       (b).  $f(x) = \sqrt{x} - \sqrt{3}x^2$        $F(x) = \frac{2}{3}x^{3/2} - \frac{\sqrt{3}}{3}x^3 + C$

9. Given that  $g'(\theta) = -\sec^2 \theta$  and  $g(\frac{\pi}{3}) = 0$ , find  $g(\theta)$ .       $g(\theta) = -\tan \theta + \sqrt{3}$

10. Given the function  $f(x) = \frac{3}{x}$ , estimate the area under the curve  $f(x)$  on the interval  $[1, 6]$  using 5 subintervals and using the right endpoint of each subinterval. [i.e. find  $R_5$ ].

$R_5 = \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] = 1 \cdot [f(2) + f(3) + f(4) + f(5) + f(6)] = 1 \cdot [3/2 + 3/3 + 3/4 + 3/5 + 3/6] = \frac{87}{20}$

11. Using the definition of the definite integral  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} R_n$ , set-up, but do not evaluate,

the summation/limit using right endpoints for the integral  $\int_0^1 x^3 + 1 dx$ .       $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[ \left( \frac{i}{n} \right)^3 + 1 \right]$

12. Section 4.3: #3

13. Evaluate the following integrals [Use integration techniques, **not** the limit definition.]:

(a).  $\int_1^2 t + 2 dt = \frac{7}{2}$       (b).  $\int_1^{x^2} t + 2 dt = \frac{1}{2}x^4 + 2x^2 - \frac{5}{2}$       (c).  $\int_0^4 \frac{x(2+x)}{\sqrt{x}} dx = \frac{352}{15}$

14. Use the Fundamental Theorem of Calculus (Part B/1) to find  $F'(x)$

(a).  $F(x) = \int_0^x t \cos t dt$        $F'(x) = x \cos x$       (b).  $F(x) = \int_{-2}^{x^2} \sqrt{t+8} dt$        $F'(x) = \sqrt{x^2+8} \cdot 2x = 2x\sqrt{x^2+8}$

15. A particle moves with a velocity of  $v(t) = -t^2 + 4t$  on the interval  $0 \leq t \leq 6$ .

(a). Find the displacement      0      (b). Find the total distance traveled       $\frac{32}{3} + \left| -\frac{32}{3} \right| = \frac{64}{3}$

16. Let  $r(t)$  be the rate at which the world's oil is consumed, where  $t$  is measured in years starting at  $t = 0$  on January 1, 2000 and  $r(t)$  is measured in barrels per year. What does  $\int_0^8 r(t) dt$  represent and what are its units? **The integral represents the change in the amount of oil consumed from 2000 to 2008. The units are barrels.**

17. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a).  $\int 3x^5 - 4x^3 + 6x + 2 dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$       (f).  $\int \sin x \cos x dx = \frac{1}{2} \sin^2 x + C$   
 (b).  $\int (3x-1)(3x^2-2x)^2 dx = \frac{1}{6}(3x^2-2x)^3 + C$       (g).  $\int \frac{5x}{\sqrt[3]{1-x^2}} dx = \frac{15}{4}(1-x^2)^{2/3} + C$   
 (c).  $\int x(3x^2-2x)^2 dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 - x^4 + C$       (h).  $\int_1^3 \frac{x^2+1}{x^2} dx = \frac{8}{3}$   
 (d).  $\int \left(1 + \frac{1}{t}\right) \left(\frac{1}{t^2}\right) dt = -\frac{1}{t} - \frac{1}{2t^2} + C$       (i).  $\int (2-x)^6 dx = -\frac{(2-x)^7}{7} + C$   
 (e).  $\int_0^{\pi/6} \sec x \tan x dx = \frac{2}{\sqrt{3}} - 1$       (j).  $\int \theta \sin(3\theta^2) d\theta = -\frac{1}{6} \cos(3\theta^2) + C$